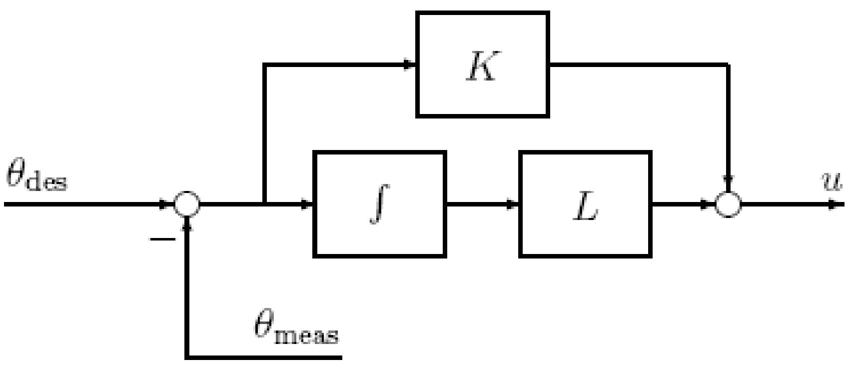
Departamentul de MECATRONICĂ
Facultatea de MECANICĂ
UNIVERSITATEA POLITHENICA TIMIŞOARA

TEORIA SISTEMELOR AUTOMATE

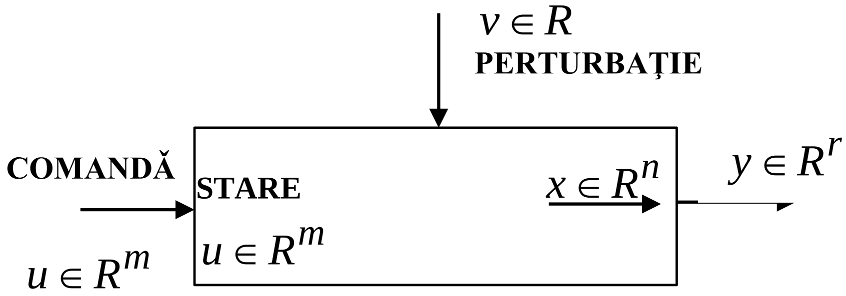


Prof. dr. ing. Valer DOLGA,

Cuprins_10
Alte metode de reprezentare dinamica a sistemelor

1. Sistem dinamic. Variabila de stare
2. Modelul de stare
3. Exemple

Sistem dinamic. Variabila de stare



$$\frac{dx}{dt} = f(x,u,v,t) \qquad \text{ecuatia diferentiala de stare}$$

$$y = g(t,x,u) \qquad \text{ecuatia de iesire}$$

→ ***modelul matematic al sistemului dinamic***

OBS.:

• un sistem - are la bază elemente între care există o serie de relaţii de dependenţă şi interacţiune;

• descrierea se face printr-un set de ecuaţii bazate pe variabilele interne ale sistemului;

• variabilele = ***variabile de stare*** ale sistemului. Expresia este sinonimă cu cea de ***starea sistemului***.

•Alegerea variabilelor de stare nu este unică.

Modelul de stare

Sisteme continue in timp

$$\frac{d\mathbf{x}}{dt} = \mathbf{F}\left[\mathbf{x}(t),\mathbf{u}(t),t\right]$$

$$\mathbf{y}(t) = \mathbf{G}\left[\mathbf{x}(t),\mathbf{u}(t),t\right]$$

$$\frac{\mathbf{dx}}{\mathbf{dt}} = \mathbf{A}\cdot\mathbf{x} + \mathbf{B}\cdot\mathbf{u} \quad \text{(ecuatia diferentiala de stare)}$$

$$\mathbf{y} = \mathbf{C}\cdot\mathbf{x} + \mathbf{D}\cdot\mathbf{u} \qquad \text{(ecuatia de iesire)}$$

- $\mathbf{A}_{\mathbf{nxn}}$ este ***matricea coeficienţilor aferentă celor “n” stări ale sistemului***;
- $\mathbf{B}_{\mathbf{nxm}}$ este ***matricea de comandă cu “m” numărul intrărilor în sistem***;
- $\mathbf{C}_{\mathbf{rxm}}$ ***matricea de ieşire cu “r” numărul de ieşiri***;
- **D** ***matricea de reacţie***;
- **x** este vectorul de stare iar **y** este vectorul de ieşire.

sistemul este cu parametri variabili în timp:

$$\frac{\mathbf{dx}}{\mathbf{dt}} = \mathbf{A}(t)\cdot\mathbf{x} + \mathbf{B}(t)\cdot\mathbf{u} \quad \textit{(ecuatia diferentida de stare)}$$

$$\mathbf{y} = \mathbf{C}(t)\cdot\mathbf{x} + \mathbf{D}(t)\cdot\mathbf{u} \quad \textit{(ecuatia de iesire)}$$

? Cum se determina modelul de stare

Ecuaţia diferenţială a sistemului este bde forma:

$$a_n y^{(n)}(t) + a_{n-1} y^{(n-1)}(t) + \ldots + a_0 y(t) = b_m u^{(m)}(t) + \ldots b_0 u(t)$$

sau

$$y^{(n)}(t) + \frac{a_{n-1}}{a_n} y^{(n-1)}(t) + \ldots + \frac{a_0}{a_n} y(t) = \frac{b_m}{a_n} u^{(m)}(t) + \ldots \frac{b_0}{a_n} u(t)$$

sau

$$y^{(n)}(t) = -\frac{a_{n-1}}{a_n} y^{(n-1)}(t) - \ldots - \frac{a_0}{a_n} y(t) + \frac{b_m}{a_n} u^{(m)}(t) + \ldots \frac{b_0}{a_n} u(t)$$

NOTATII

$$x_1(t) = y(t)$$

$$x_2(t) = \frac{dx_1(t)}{dt} = \frac{dy(t)}{dt}$$

$$\ldots$$

$$x_n(t) = x_{n-1}^{(n-1)}(t) = y^{(n-1)}(t)$$

si

$$\frac{dx_n(t)}{dt} = y^{(n)}(t)$$

Ecuatia diferentiala a sistemului

$$\frac{dx_1(t)}{dt} = x_2(t)$$

$$\frac{dx_2(t)}{dt} = x_3(t)$$

$$\ldots$$

$$\frac{dx_n(t)}{dt} = -\frac{a_{n-1}}{a_n}x_n - \frac{a_{n-2}}{a_n}x_{n-1} - \ldots - \frac{a_0}{a_n}x_1 + \frac{b_m}{a_n}u^{(m)}(t) + \ldots \frac{b_0}{a_n}u(t)$$

$$\begin{bmatrix} \frac{dx_1(t)}{dt} \\ \frac{dx_2(t)}{dt} \\ \frac{dx_3(t)}{dt} \\ .. \\ \frac{dx_n}{dt}(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & .... & 0 \\ 0 & 0 & 1 & ... & 0 \\ & & & ... & \\ & & & ... & \\ \frac{a_0}{a_n} & \frac{a_1}{a_n} & \frac{a_2}{a_n} & ... & \frac{a_{n-2}}{a_n} \end{bmatrix} \cdot \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \\ .. \\ x_n(t) \end{bmatrix} + \mathbf{B} \cdot \mathbf{u}$$

Exemplu – sistem de ordinul 2

$$a_2 \frac{d^2y}{dt^2} + a_1 \frac{dy}{dt} + a_0 y = b_0 u$$

$$\frac{d^2y}{dt^2} + \frac{a_1}{a_2}\frac{dy}{dt} + \frac{a_0}{a_2} y = \frac{b_0}{a_2} u$$

$$\frac{d^2y}{dt^2} = -\frac{a_1}{a_2}\frac{dy}{dt} - \frac{a_0}{a_2} y + \frac{b_0}{a_2} u$$

$$x_2 = \frac{dx_1}{dt} = \frac{dy}{dt}$$

$$\frac{d^2y}{dt^2} = -\frac{a_1}{a_2}\frac{dy}{dt} - \frac{a_0}{a_2}y + \frac{b_0}{a_2}u$$

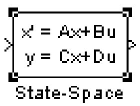
$$\frac{dx_2}{dt} = \frac{d^2y}{dt^2} \qquad x_1 = y$$

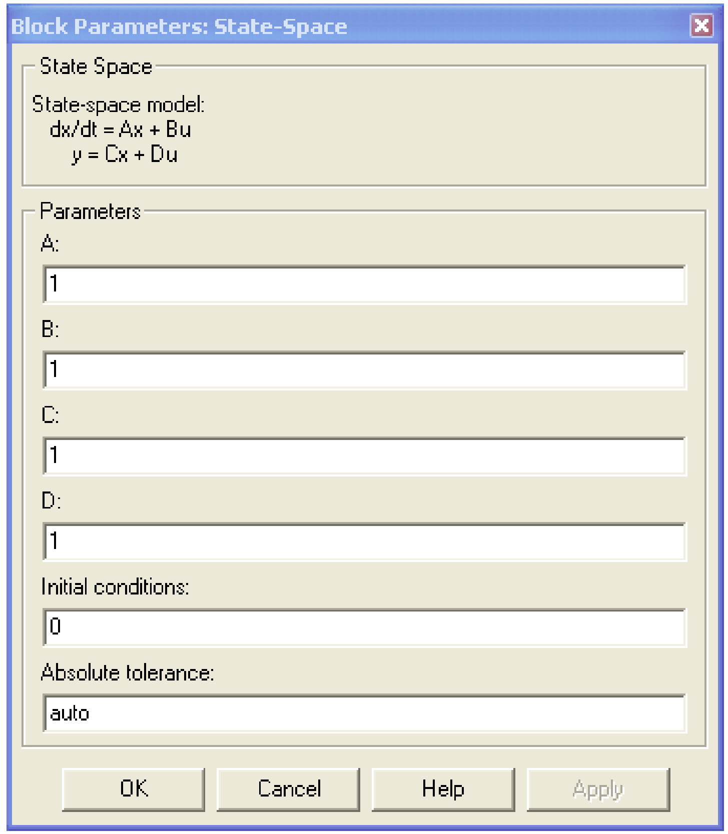
$$\begin{cases} \dfrac{dx_1}{dt} = 0 \cdot x_1 + 1 \cdot x_2 + 0 \cdot u \\ \dfrac{dx_2}{dt} = -\dfrac{a_0}{a_2}x_1 - \dfrac{a_1}{a_2}x_2 + \dfrac{b_0}{a_2}u \end{cases}$$

$$\begin{bmatrix} \dfrac{dx_1}{dt} \\ \dfrac{dx_2}{dt} \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} b_{11} \\ b_{21} \end{bmatrix} [u]$$

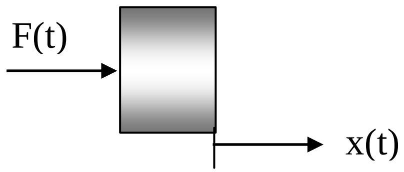
$$A = \begin{bmatrix} 0 & 1 \\ -\dfrac{a_0}{a_2} & -\dfrac{a_1}{a_2} \end{bmatrix} \qquad B = \begin{bmatrix} 0 \\ \dfrac{b_0}{a_2} \end{bmatrix}$$

*Matlab / Simulink / Continuous / **State – Space***





Exemplu



M= 10 kg

F= 100 N

$$m \cdot \frac{d^2x}{dt^2} = F$$

$$x_1 = x$$

$$x_2 = \frac{dx_1}{dt} = \frac{dx}{dt}$$

$$\frac{dx_2}{dt} = \frac{d^2x}{dt^2} = \frac{F}{m}$$

$$\begin{cases} \begin{bmatrix} \frac{dx_1}{dt} \\ \frac{dx_2}{dt} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{m} \end{bmatrix} \cdot [F] \\ [y] = [1 \quad 0] \cdot \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + [0] \cdot [F] \end{cases}$$

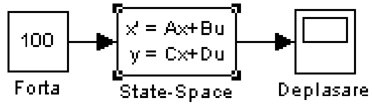
Cazul I

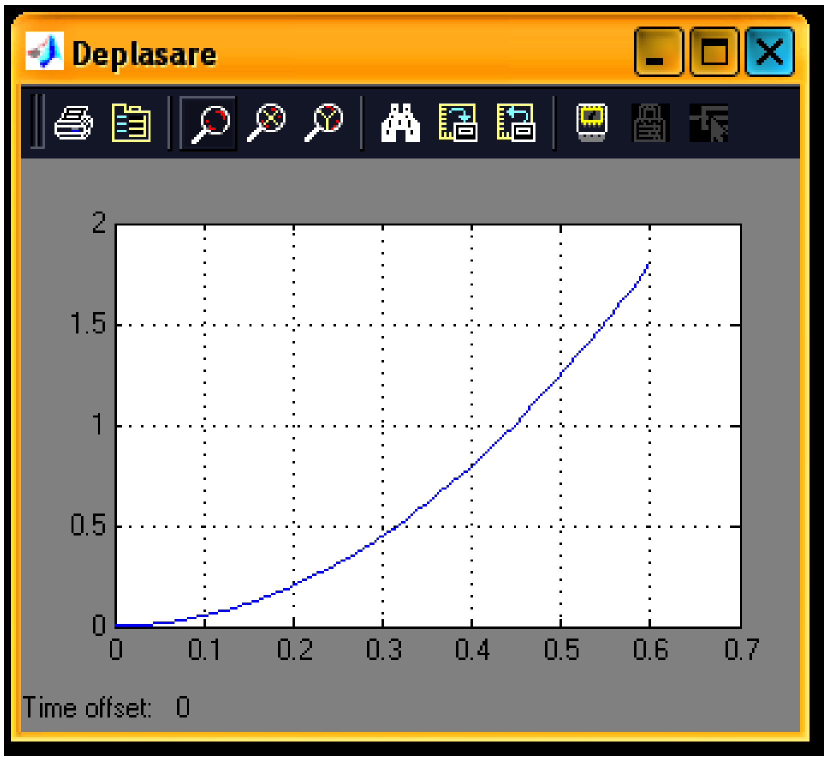
sau

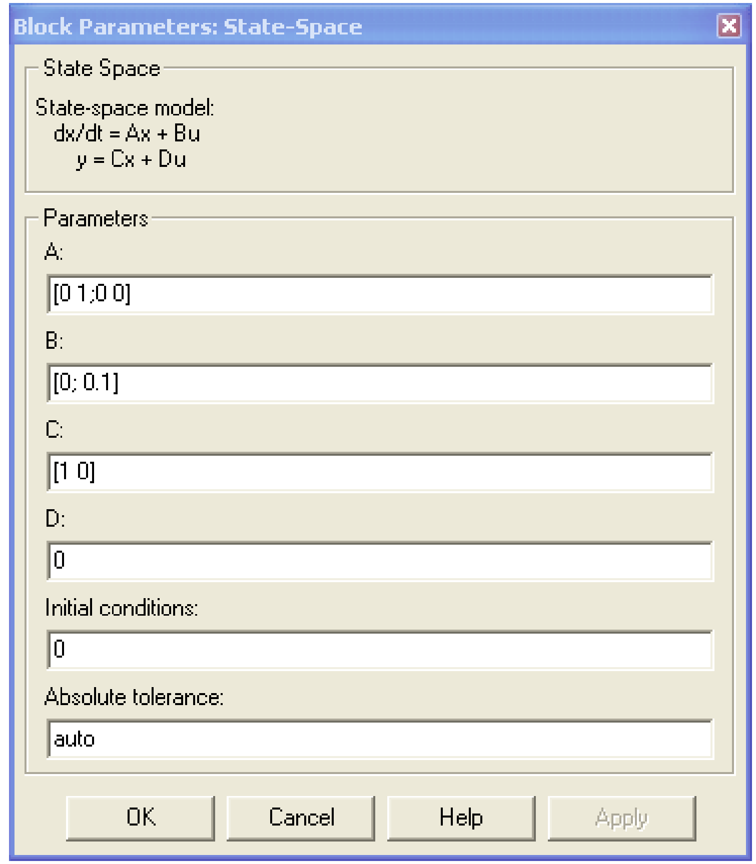
$$[y] = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix} \cdot [F]$$

Cazul II

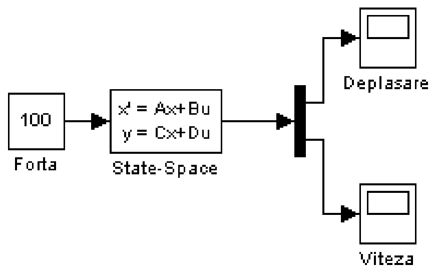
Cazul 1:

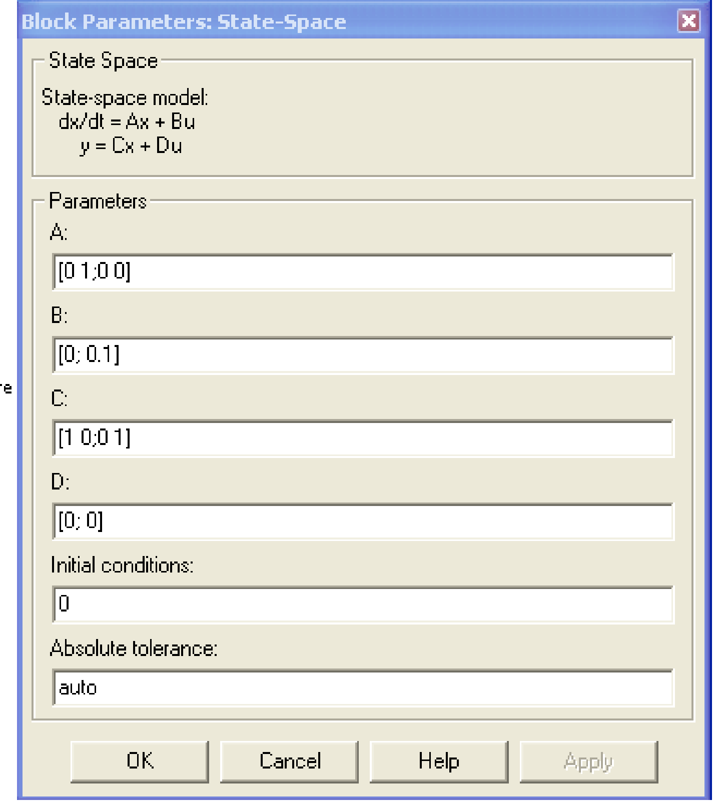

100
Forta
x' = Ax+Bu
y = Cx+Du
State-Space
Deplasare

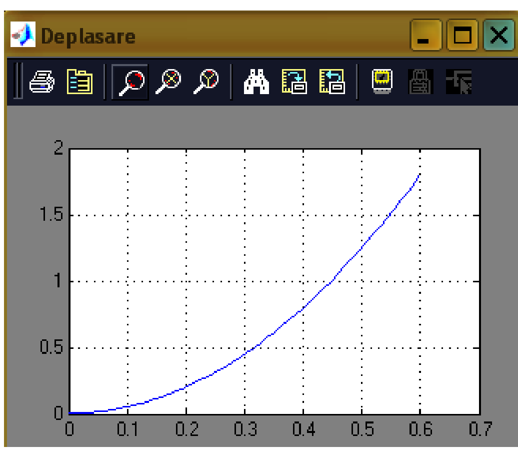

Deplasare
2
1.5
1
0.5
0
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
Time offset: 0

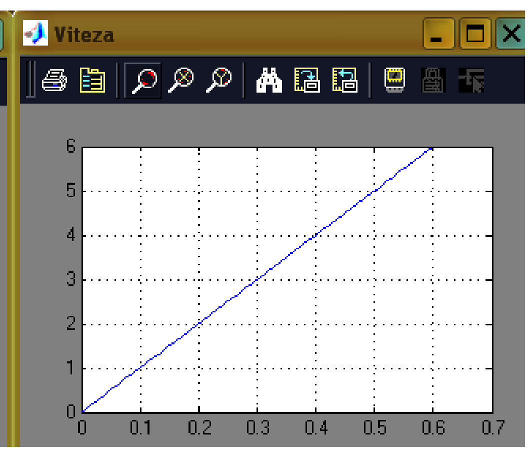

Block Parameters: State-Space
State Space
State-space model:
dx/dt = Ax + Bu
y = Cx + Du
Parameters
A:
[0 1;0 0]
B:
[0; 0.1]
C:
[1 0]
D:
0
Initial conditions:
0
Absolute tolerance:
auto
OK
Cancel
Help
Apply

Cazul 2


100
Forta
x' = Ax+Bu
y = Cx+Du
State-Space
Deplasare
Viteza


Block Parameters: State-Space
State Space
State-space model:
dx/dt = Ax + Bu
y = Cx + Du
Parameters
A:
[0 1;0 0]
B:
[0; 0.1]
C:
[1 0;0 1]
D:
[0; 0]
Initial conditions:
0
Absolute tolerance:
auto
OK
Cancel
Help
Apply


Deplasare
2
1.5
1
0.5
0
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7


Viteza
6
5
4
3
2
1
0
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7