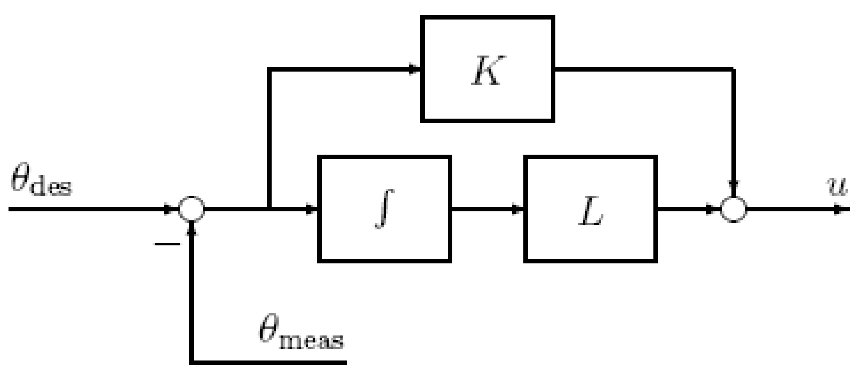
Departamentul de MECATRONICĂ
Facultatea de MECANICĂ
UNIVERSITATEA POLITEHNICA TIMIŞOARA

TEORIA SISTEMELOR AUTOMATE

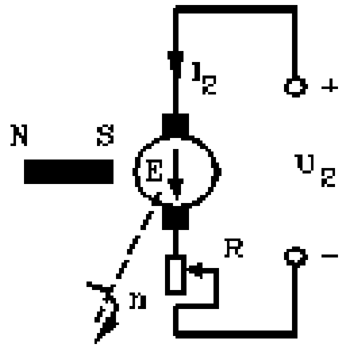


Prof. dr. ing. Valer DOLGA,

Cuprins_5
Metode pentru analiza dinamica a sistemelor

1. Metoda integrarii ecuatiei diferentiale
2. Metoda transformatei Laplace
3. Transformata Laplace
4. Inversa transformatei Laplace

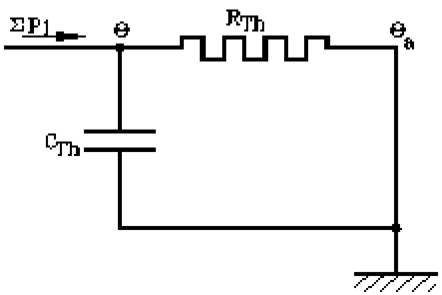
....modelare



$$u_2 = e + \left(R_A + R\right)\cdot i_2 + L_A \cdot \frac{d i_2}{d t}$$

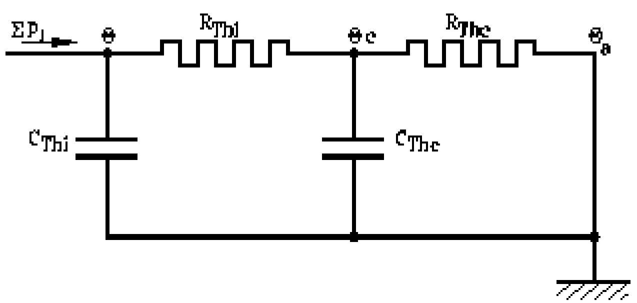
$$\frac{\partial}{\partial t}\left(\frac{\partial E_c}{\partial \dot{q}_k}\right) - \frac{\partial E_c}{\partial q_k} = Q_k$$

$$J_t \cdot \varepsilon_r + \frac{2}{i^2}\cdot m_3 \cdot \omega_r \cdot r_3 \cdot v_3 = M_m - \frac{M_f}{i} - M_{fs} - M_{fv}$$



$$\sum p_i = C_{Th}\frac{d\theta}{d t} + \frac{\theta - \theta_a}{R_{Th}}$$

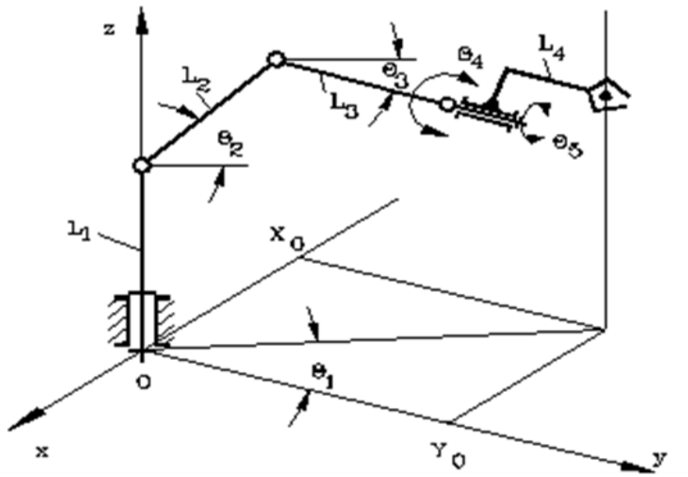
$$\Delta\theta = \left(\sum p_i\right)\cdot R_{Th}\cdot\left(1 - e^{-\frac{t}{\tau_{Th}}}\right)$$

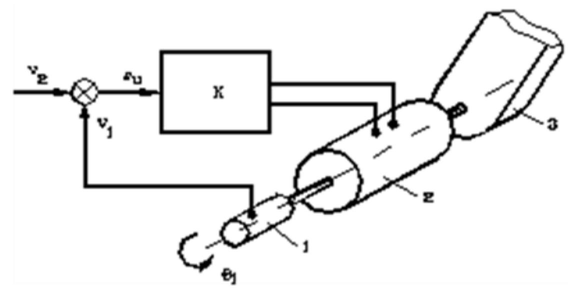


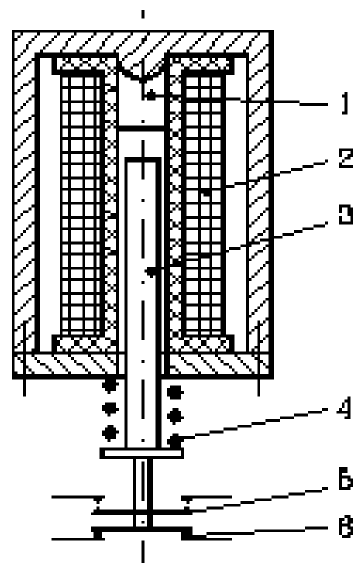
$$\sum p_i = C_{Thi}\cdot\frac{d\theta_i}{d t} + \frac{\theta_i - \theta_c}{R_{Thi}}$$

$$\frac{\theta_i - \theta_c}{R_{Thi}} = C_{Thc}\cdot\frac{d\theta}{d t} + \frac{\theta_c - \theta_a}{R_{Thc}}$$

...

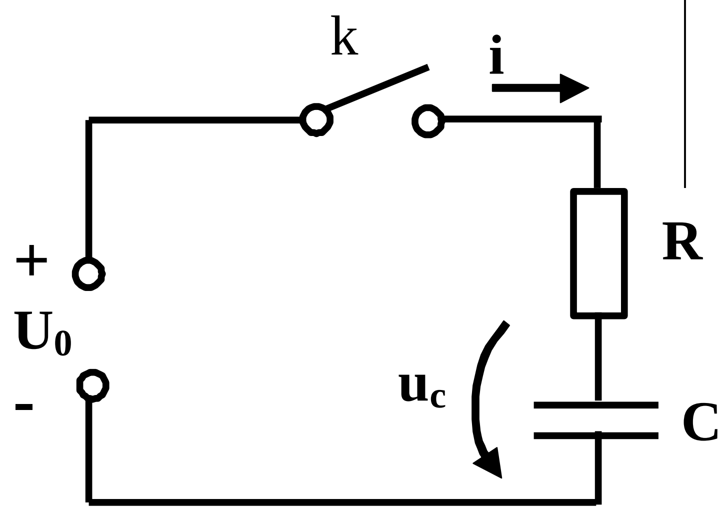

z
L2
θ3
θ4
L4
L3
θ5
θ2
L1
X0
O
θ1
x
Y0
y


X
3
2
1
θ1


1
2
3
4
5
6

Metoda integrarii ecuatiei

$$RC\frac{du_c}{dt} + u_c = U_0$$



$$a_1\frac{dy(t)}{dt} + a_0 y(t) = bx(t)$$

$$y(t) = y_o(t) + y_p(t)$$

$y_o(t)$ – este soluţia pentru ecuaţia omogenă

$$\sum_{k=0}^{n} a_k \frac{d^k y(t)}{dt^k} = 0$$

$$\sum_{k=0}^{n} a_k s^k = 0$$

$$y_o(t) = C_1 e^{s_1 t} + C_2 e^{s_2 t} + \ldots.$$

$$RC \cdot s + 1 = 0$$

$$s = -\frac{1}{RC}$$

$$u_{C0} = C_1 \cdot e^{-\frac{t}{RC}}$$

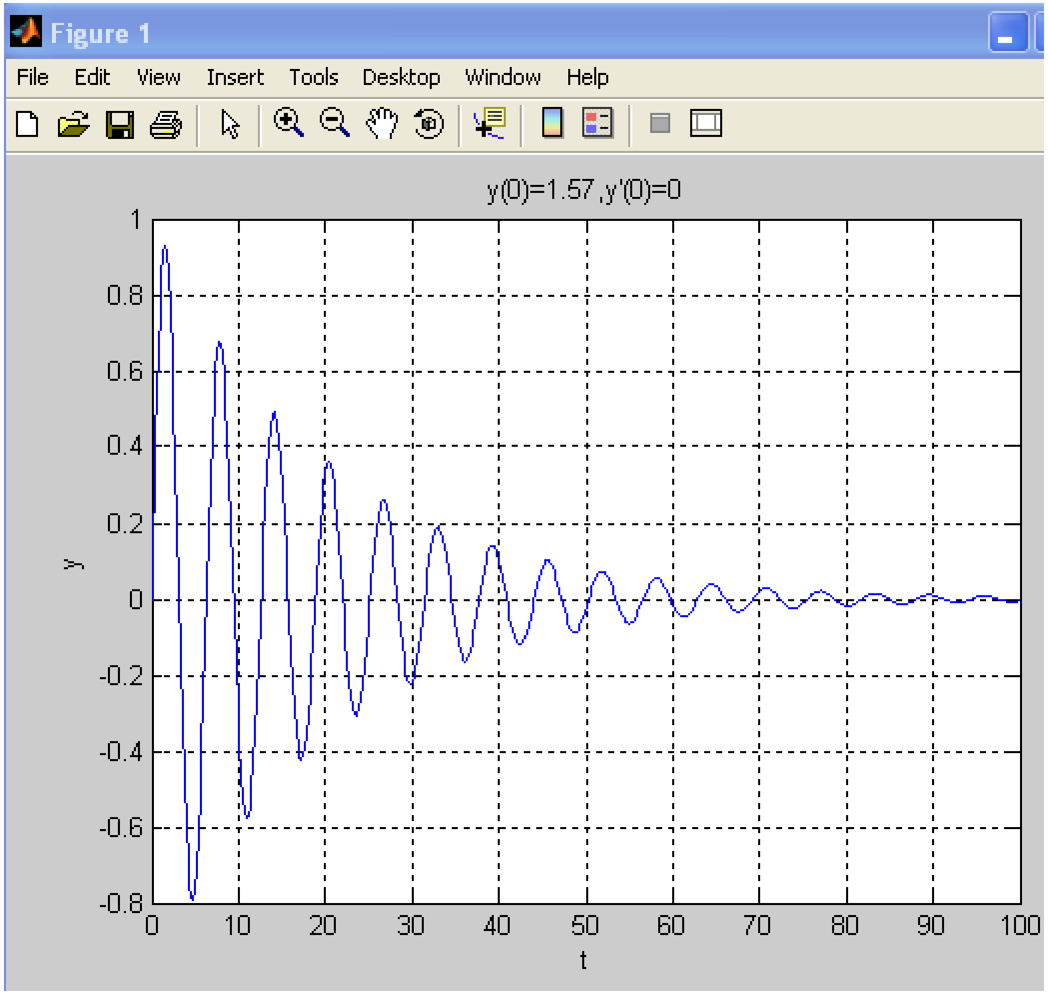
yp(t) – este o soluţie particulară a ecuatiei neomogene $u_{Cp} = U_0$

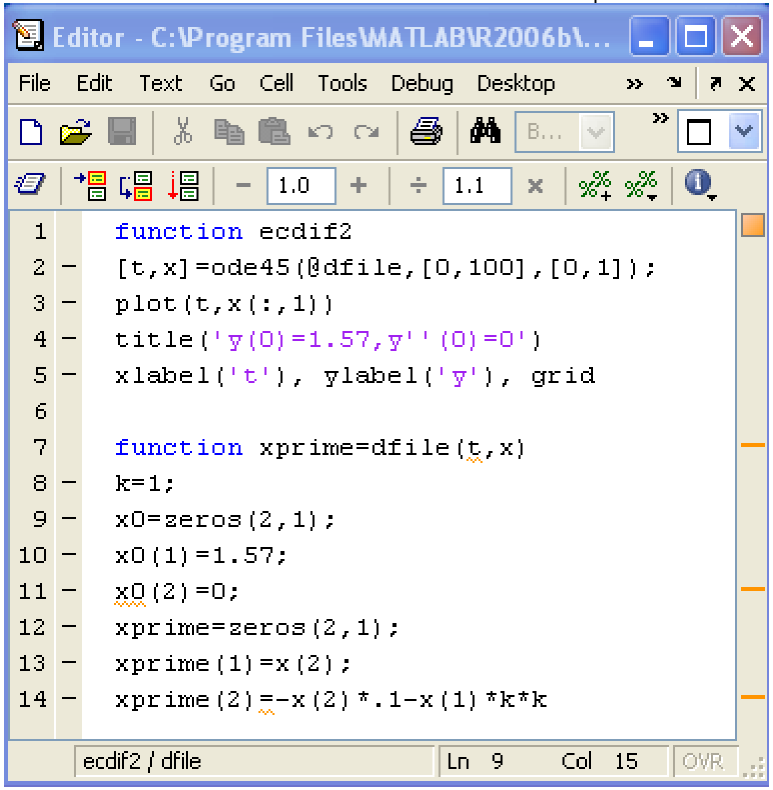
$$u_C = u_{C0} + u_{Cp} = C_1 \cdot e^{-\frac{t}{RC}} + U_0$$

Constanta C_1- pentru momentul t = 0: $u_C = 0$ → $C_1 = -U_0$

$$u_C = U_0\left(1 - e^{-\frac{t}{RC}}\right)$$

Rezolvare - Matlab



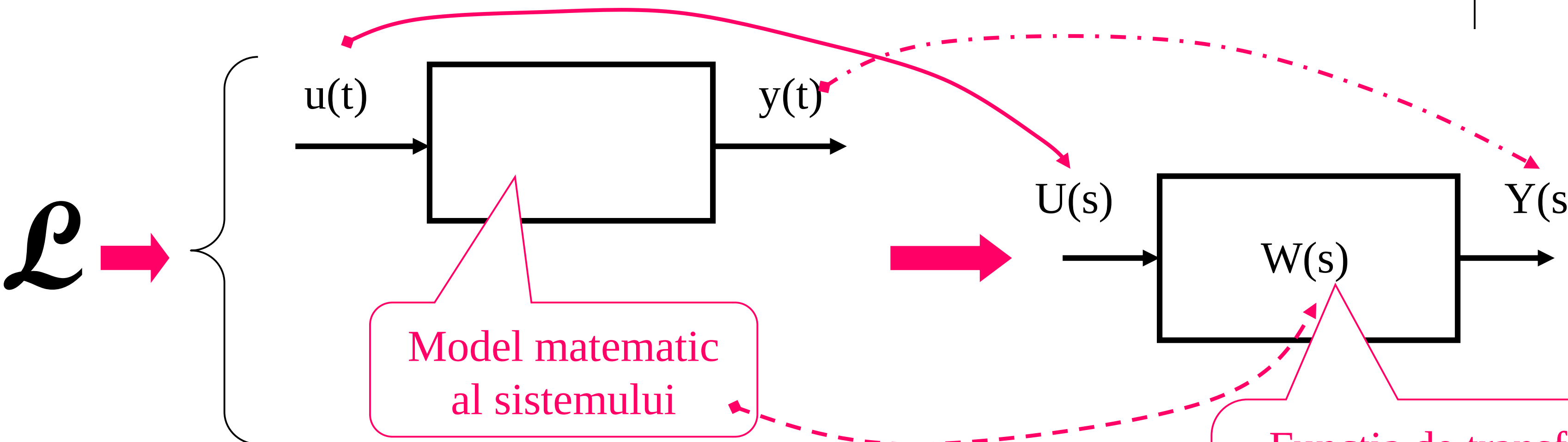


Metoda transformatei Laplace

Ecuatii diferentiale functie de timp: f(t)

$$\mathcal{L}\{f(t)\}$$

Ecuatii algebrice in "s": F(s)

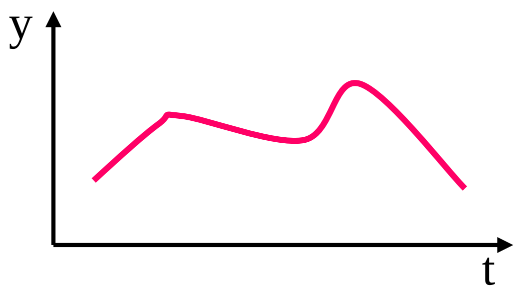


Functia de transfer

$$W(s)=\frac{Y(s)}{U(s)}$$

$$Y(s)=U(s)\cdot W(s)$$

$$\mathcal{L}^{-1}\{Y(s)\}$$



$$\equiv y(t)$$

Transformata Laplace

$$\int_0^{\infty} \left| f(t) e^{-\alpha t} \right| dt < \infty \qquad \alpha \in R, 0 < \alpha < \infty$$

$$\mathcal{L}\{f(t)\} = \int_0^{\infty} f(t) e^{-st} dt = F(s)$$

$\mathcal{L}$ = operatorul Laplace

$s = \sigma + j\omega$

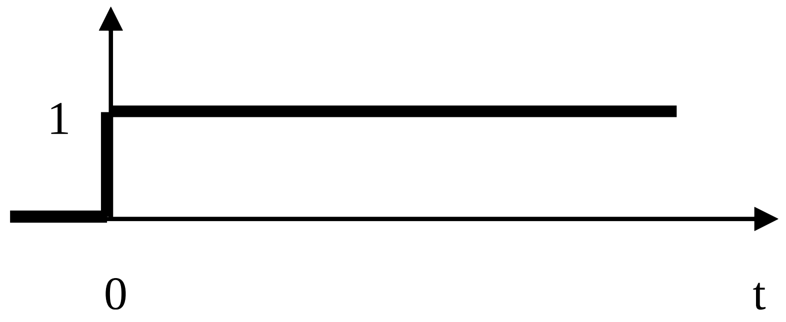
Semnale standard:

$u(t) \equiv$

- •semnal treapta unitara
- •semnal rampa unitara
- •Semnal sinusoidal
- •Semnal cosinusoidal …….

Funcţia unitate (treaptă unitară)

$$f(t) = \begin{cases} = 0 & t < 0 \\ = 1 & t \geq 0 \end{cases}$$



$$f(t) = k$$ -functia constanta, pentru $t \geq 0$

$$\mathcal{L}\{f(t)\} = F(s) = \int_0^\infty f(t) e^{-st} dt = \int_0^\infty e^{-st} dt$$

$$\int e^u du = e^u$$

$$-st = u$$

$$t = -\frac{1}{s} u$$

$$\mathcal{L}\{f(t)\}=F(s)=\int_0^\infty e^u dt=\int_0^\infty e^u d(-\frac{1}{s}u)=-\frac{1}{s}(-\int_{-\infty}^0 e^u du)=\frac{1}{s}e^u\Big|_{-\infty}^0=\frac{1}{s}(1-0)=$$

$$=\frac{1}{s}$$

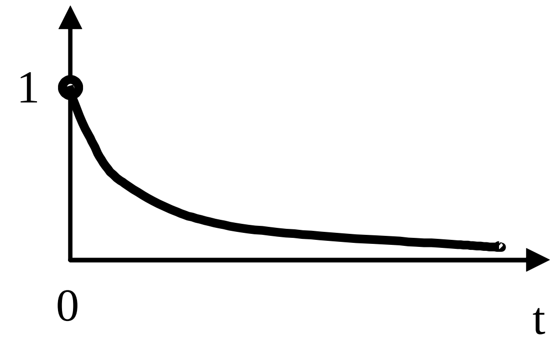
$$\mathcal{L}\{1\}=\frac{1}{s}$$

$$\mathcal{L}\{k\}=\mathcal{L}\{k\cdot 1\}=k\cdot\mathcal{L}\{1\}=k\frac{1}{s}$$

$$\mathcal{L}\{f(t)\}=F(s)=\int_0^\infty f(t)e^{-st}dt=\int_0^\infty k\cdot e^{-st}dt=k\cdot\int_0^\infty e^{-st}dt=k\cdot\frac{1}{s}$$

Functia exponentiala

$$f(t)=\begin{cases}0, \; pentru\ t<0\\ e^{-\alpha t}, \; pentru\ t\geq 0\end{cases}$$



$$\mathcal{L}\{f(t)\}=F(s)=\int_0^\infty f(t)e^{-st}dt=\int_0^\infty e^{-\alpha t}e^{-st}dt=\int_0^\infty e^{-(s+\alpha)t}dt=$$

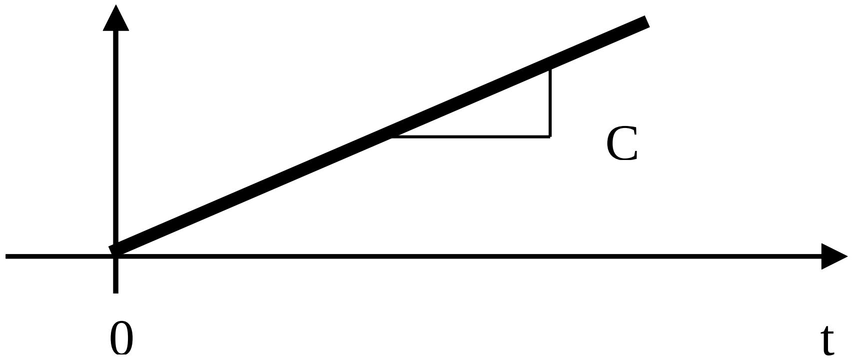
$$=-\frac{1}{s+\alpha}e^{-(s+\alpha)t}\Big|_0^\infty=0-\left(-\frac{1}{s+\alpha}\right)=\frac{1}{s+\alpha}$$

$$\mathcal{L}\left\{e^{-\alpha t}\right\}=\frac{1}{s+\alpha}$$

Functia rampa

$$f(t)=\begin{cases}=0 & t<0\\ =Ct & t\geq 0\end{cases}$$

C este o constantă reală



$$\mathcal{L}\{f(t)\}=F(s)=\int_0^\infty f(t)e^{-st}dt=\int_0^\infty Cte^{-st}dt=C\cdot\int_0^\infty te^{-st}dt$$

Integrarea prin parti

$$\begin{cases}\int udv=uv-\int vdu\\ \int_a^b udv=uv\Big|_a^b-\int_a^b vdu\end{cases}$$

$$u=t$$

$$du=dt$$

$$v=-\frac{1}{s}e^{-st}$$

$$dv=e^{-st}dt$$

$$F(s)=C\int_0^\infty te^{-st}dt=C\cdot(uv\Big|_0^\infty-\int_0^\infty vdu)=C\cdot\left\{-\frac{t}{s}e^{-st}\Big|_0^\infty-\int_0^\infty\left(-\frac{1}{s}e^{-st}C\right)dt\right\}=$$

$$=0+\frac{C}{s}\int_0^\infty e^{-st}dt=\frac{C}{s}\left(-\frac{1}{s}\right)e^{-st}\Big|_0^\infty=\frac{C}{s^2}$$

$$\mathcal{L}\{Ct\}=\frac{C}{s^2}$$

identitatea lui Euler

$$e^{\pm j\alpha} = \cos\alpha \pm j\sin\alpha$$

$$j = \sqrt{-1}$$

$$\left.\begin{aligned} e^{+j\alpha} &= \cos\alpha + j\sin\alpha \\ e^{-j\alpha} &= \cos\alpha - j\sin\alpha \end{aligned}\right\} \xrightarrow{+}$$

$$\cos\alpha = \frac{e^{j\alpha} + e^{-j\alpha}}{2}$$

$$sau\ daca\ \alpha = \omega t$$

$$\cos\omega t = \frac{e^{j\omega t} + e^{-j\omega t}}{2}$$

$$\left.\begin{aligned} e^{+j\alpha} &= \cos\alpha + j\sin\alpha \\ e^{-j\alpha} &= \cos\alpha - j\sin\alpha \end{aligned}\right\} \xrightarrow{-}$$

$$\sin\alpha = \frac{e^{j\alpha} - e^{-j\alpha}}{2j}$$

$$sau\ daca\ \alpha = \omega t$$

$$\sin\omega t = \frac{e^{j\omega t} - e^{-j\omega t}}{2j}$$

Functia “cos”

$$\mathcal{L}\{f(t)\} = F(s) = \int_0^\infty f(t)e^{-st}dt = \int_0^\infty e^{-st}\cos(\omega t)dt =$$

$$= \int_0^\infty e^{-st}\left[\frac{e^{j\omega t} + e^{-j\omega t}}{2}\right]dt = \frac{1}{2}\int_0^\infty \left[e^{(-s+j\omega)t} + e^{-(s+j\omega)t}\right]dt$$

$$s = \sigma + j\omega$$

$$\mathcal{L}\{f(t)\} = F(s) = \frac{1}{2}\int_0^\infty (e^{(-\sigma - j\omega + j\omega)t} + e^{-\sigma - j\omega - j\omega)t})dt =$$

$$= \frac{1}{2}\int_0^\infty e^{-\sigma t}dt + \frac{1}{2}\int_0^\infty e^{-(\sigma+2j\omega)t}dt = -\frac{1}{2\sigma}e^{-\sigma t}\Big|_0^\infty - \frac{1}{2(\sigma+2j\omega)}e^{-(\sigma+2j\omega)t}\Big|_0^\infty =$$

$$= -\frac{1}{2}(-\frac{1}{\sigma} + -\frac{1}{\sigma+2j\omega}) = \frac{2\sigma+2j\omega}{2\sigma(\sigma+2j\omega)} = \frac{\sigma+j\omega}{\sigma^2+2\sigma j\omega} = \frac{s}{s^2+\omega^2}$$

$$\mathcal{L}\{\cos(\omega t)\} = \frac{s}{s^2+\omega^2}$$

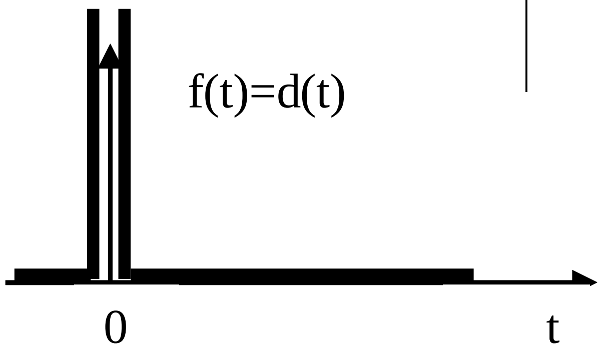
Functia “sin”

Studiu individual

?

Functia impuls unitar (functia Dirac)

$$\begin{cases} f(t) = \delta(t) = \begin{cases} 0, & pentru\ t \neq 0 \\ \infty, & pentru\ t = 0 \end{cases} \\ \int_{-\infty}^{\infty} \delta(t)dt = 1 \end{cases}$$

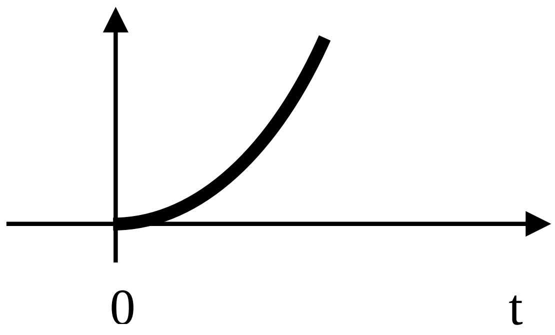


$$\mathcal{L}\{f(t)\} = F(s) = \int_{0}^{\infty} \delta(t)e^{-st}dt = 1$$

$$\mathcal{L}\{\delta(t)\} = 1$$

Functia patratica

$$f(t)=\begin{cases}0 & pentru\ t<0\\ Ct^2 & pentru\ t\geq 0\end{cases}$$



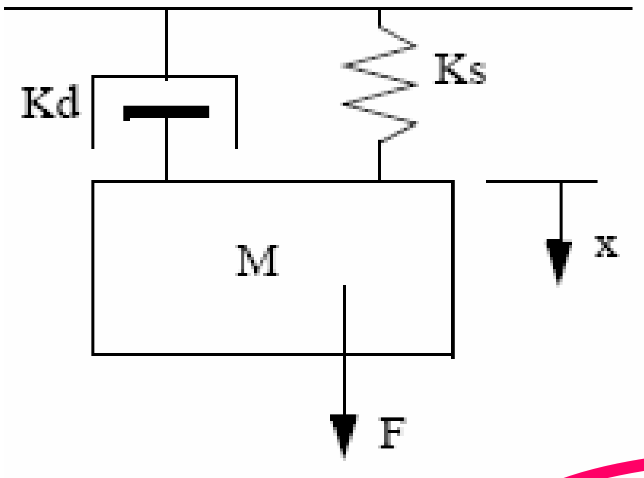
- integrare prin parti $\Rightarrow$ $$\mathcal{L}\left\{Ct^2\right\}=\frac{2C}{s^3}$$

- generalizare $\Rightarrow$ $$\mathcal{L}\left\{t^n\right\}=\frac{n!}{s^{n+1}}$$

Transformata Laplace – proprietati (extras)

NR. CRT.	PROPRIE-TATEA	EXPRIMAREA ÎN FUNCŢIE DE TIMP	TRANSFORMATA LAPLACE
1	Liniaritate	$\mathcal{L}[ax(t)]$ $\mathcal{L}[x_1(t)+x_2(t)]$ $\mathcal{L}[\sum a_k x_k(t)]$	$aF(s)$ $F_1(s)+F_2(s)$ $\sum a_k \cdot F_k(s)$
2	Derivare în funcţie de timp	$\mathcal{L}\left[\frac{dx(t)}{dt}\right]$ $\mathcal{L}\left[\frac{d^2x(t)}{dt^2}\right]$	$sF(s)-x(0)$ $s^2F(s)-sx(0)-x'(0)$
3	Derivare de ordinul "n" în funcţie de timp	$\mathcal{L}\left[\frac{d^nx(t)}{dt^n}\right]$	$s^nF(s)-s^{n-1}x(0)-s^{n-2}x^{(1)}(0)-$ $\ldots-x^{(n-1)}(0)$

Exemplu



- forţa elastică F_e; - forţa de amortizare F_a;

- forţa de inerţie F_i

$$F(t) = F_e + F_a + F_i$$

$$F(t) = K_s \cdot x(t) + K_d \cdot \frac{dx}{dt} + M \cdot \frac{d^2x}{dt^2}$$

$$\mathcal{L}\{f(t)\} = \mathcal{L}\left\{K_s x + K_d \cdot \frac{dx}{dt} + M \cdot \frac{d^2x}{dt^2}\right\} =$$

$$= \mathcal{L}\{K_s x\} + \mathcal{L}\left\{K_d \cdot \frac{dx}{dt}\right\} + \mathcal{L}\left\{M \cdot \frac{d^2x}{dt^2}\right\} =$$

$$= K_s \cdot X(s) + K_d \cdot sX(s) + M \cdot s^2 X(s) =$$

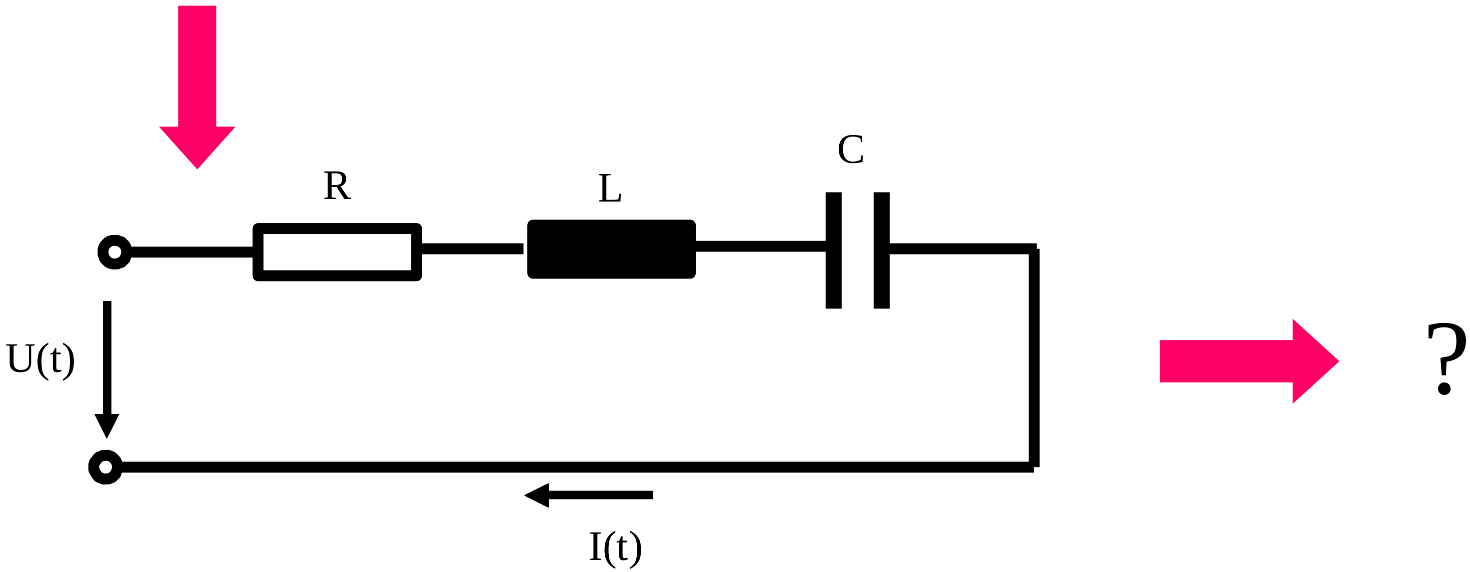
$$= \left(K_s + K_d \cdot s + M \cdot s^2\right) X(s)$$

$$\mathcal{L}\{F(t)\} = F(s)$$

$$W(s) = \frac{X(s)}{F(s)} = \frac{1}{Ms^2 + K_d s + K_s}$$

Exemplu – domeniul electric

rezistor	$U(t) = R \cdot I(t)$	$U(s) = R \cdot I(s)$
inductivitate	$U(t) = L \cdot \dfrac{dI(t)}{dt}$	$U(s) = sLI(s)$
capacitate	$U(t) = \dfrac{1}{C}\int I(t)dt$	$U(s) = \dfrac{1}{C} \cdot \dfrac{I(s)}{s}$



Anexa – transformata Laplace (extras)

NR. CRT.	PROPRIETATEA	FUNCŢIA DE TIMP	TRANSFORMATA LAPLACE
1	Funcţia Dirac	$f(t)=\delta(t)=\begin{cases}0, & pentru\ t\neq 0\\ \infty, & pentru\ t=0\end{cases}$	$\mathcal{L}\{\delta(t)\}=1$
2	Funcţia unitate (treaptă unitară)	$f(t)=\begin{cases}=0 & t<0\\ =1 & t\geq 0\end{cases}$	$\mathcal{L}\{1\}=\frac{1}{s}$
3	Funcţia rampă	$f(t)=\begin{cases}=0 & t<0\\ =Ct & t\geq 0\end{cases}$	$\mathcal{L}\{Ct\}=\frac{C}{s^2}$
4	Funcţia rampă pătratică	$f(t)=\begin{cases}0 & pentru\ t<0\\ Ct^2 & pentru\ t\geq 0\end{cases}$	$\mathcal{L}\{Ct^2\}=\frac{2C}{s^3}$
5	Funcţia rampă de ordinul n	$f(t)=\begin{cases}0 & pentru\ t<0\\ Ct^n & pentru\ t\geq 0\end{cases}$	$\mathcal{L}\{t^n\}=\frac{n!}{s^{n+1}}$
6	Funcţia exponenţială	$f(t)=\begin{cases}0, & pentru\ t<0\\ e^{-\alpha t}, & pentru\ t\geq 0\end{cases}$	$\mathcal{L}\{e^{-\alpha t}\}=\frac{1}{s+\alpha}$
7	Funcţia periodică armonică	$f(t)=\cos\omega t$	$\mathcal{L}\{\cos(\omega t)\}=\frac{s}{s^2+\omega^2}$

Inversa transformatei Laplace

Functie in "s" → ? → Functie de timp

$$f(t) = \mathcal{L}^{-1}\big(F(s)\big)$$

$$F(s) = Z_1(s) + Z_2(s) + \ldots. + Z_n(s)$$

↓

$$f(t) = \mathcal{L}^{-1}\big(F(s)\big) = \mathcal{L}^{-1}\{Z_1(s) + Z_2(s) + \ldots Z_n(s)\} =$$

$$= \sum_{i=1}^{n} \mathcal{L}^{-1}\{Z_i(s)\}$$

$$Y(s) = \frac{Q(s)}{P(s)}$$

gr [Q(s)]= m

gr [P(s)] = n

Daca m >= n

$$Y(s) = \frac{Q(s)}{P(s)} = K(s) + \frac{N(s)}{P(s)}$$

$$y(t) = \mathcal{L}^{-1}\left(Y(s)\right) = \mathcal{L}^{-1}\left\{K(s) + \frac{N(s)}{P(s)}\right\} =$$

$$= \mathcal{L}^{-1}\{K(s)\} + \mathcal{L}^{-1}\left\{\frac{N(s)}{P(s)}\right\}$$

Numitorul P(s):

a) Radacini reale si distincte: $r_1, r_2, \ldots, r_n$

b) Radacini reale, distincte si multiple

c) Radacini complexe

a)

$$\frac{N(s)}{P(s)} = \frac{N(s)}{(s+r_1)(s+r_2)\ldots(s+r_n)} = \frac{k_1}{s+r_1} + \frac{k_2}{s+r_2} + \ldots \frac{k_n}{s+r_n}$$

$$k_i = (s+r_1)\frac{N(s)}{P(s)}\bigg|_{s=-r_i}$$

$$\mathcal{L}\left\{e^{-\alpha t}\right\} = \frac{1}{s+\alpha}$$

$$\mathcal{L}^{-1}\left(\frac{N(s)}{P(s)}\right) = \mathcal{L}^{-1}\left\{\frac{k_1}{s+r_1}\right\} + \mathcal{L}^{-1}\left\{\frac{k_2}{s+r_2}\right\} + \ldots\ldots\mathcal{L}^{-1}\left\{\frac{k_n}{s+r_n}\right\} =$$

$$= k_1 e^{-r_1 t} + k_2 e^{-r_2 t} + \ldots\ldots + k_n e^{-r_n t}$$

Exemplu – pentru cazul “a”

$$X(s)=\frac{1}{s^3+3s^2+2s}=\frac{1}{\left(s^2+3s+2\right)s}=\frac{1}{s(s+1)(s+2)}$$

$$s_{1,2}=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$X(s)=\frac{1}{s(s+1)(s+2)}=\frac{A}{s}+\frac{B}{s+1}+\frac{C}{s+2}$$

$$sX(s)=A+s\cdot\frac{B}{s+1}+s\cdot\frac{C}{s+2} \Rightarrow sX(s)\Big|_{s=0}=A$$

$$A=sX(s)\Big|_{s=0}=s\frac{1}{s(s+1)(s+2)}\Bigg|_{s=0}=\frac{1}{2}$$

$$B=(s+1)X(s)\Big|_{s=-1}=(s+1)\frac{1}{s(s+1)(s+2)}\Big|_{s=-1}=$$

$$=\frac{1}{s(s+2)}\Big|_{s=-1}=\frac{1}{(-1)(-1+2)}=-1$$

$$C=(s+2)X(s)\Big|_{s=-2}=(s+2)\frac{1}{s(s+1)(s+2)}\Big|_{s=-2}=\frac{1}{s(s+1)}\Big|_{s=-2}$$

$$=\frac{1}{(-2)(-2+1)}=\frac{1}{2}$$

$$x(t)=\mathcal{L}^{-1}\left\{\frac{1}{2}\cdot\frac{1}{s}\right\}-\mathcal{L}^{-1}\left\{\frac{1}{s+1}\right\}+\mathcal{L}^{-1}\left\{\frac{1}{2}\cdot\frac{1}{s+2}\right\}=$$

$$=\frac{1}{2}-e^{-t}+\frac{1}{2}\cdot e^{-2t}$$

Cazul - b

$$Z(s)=\frac{N(s)}{P(s)}=\frac{N(s)}{(s+r_1)(s+r_2)\ldots(s+r_{n-p})(s+r_{n-p+1})^p}=$$

$$=\frac{k_1}{s+r_1}+\ldots\frac{k_n}{s+r_{n-p}}+\frac{A_1}{(s+r_{n-p+1})}+\frac{A_2}{(s+r_{n-p+1})^2}+\ldots+\frac{A_p}{(s+r_{n-p+1})^p}$$

$$A_p=\left[(s+r_{n-p+1})^p Z(s)\right]_{s=-r_{n-p+1}}$$

$$A_{p-1}=\frac{d}{ds}\left[(s+r_{n-p+1})^p Z(s)\right]_{s=-r_{n-p+1}}$$

$$\ldots\ldots\ldots\ldots\ldots\ A_1=\frac{1}{(p-1)!}\frac{d^{p-1}}{ds^{p-1}}\left[(s+r_{n-p+1})^p Z(s)\right]_{s=-r_{n-p+1}}$$

Exemplu-cazul b

$$Y(s)=\frac{1}{s^3(s+1)(s+2)}=\frac{k_1}{s+1}+\frac{k_2}{s+2}+\frac{A_1}{s}+\frac{A_2}{s^2}+\frac{A_3}{s^3}$$

$$k_1=(s+1)\cdot\left.\frac{1}{s^3(s+1)(s+2)}\right|_{s=-1}=-1$$

$$k_2=(s+2)\cdot\left.\frac{1}{s^3(s+1)(s+2)}\right|_{s=-2}=\frac{1}{8}$$

$$A_3=\left.\left[s^3\cdot\frac{1}{s^3(s+1)(s+2)}\right]\right|_{s=0}=\frac{1}{2}$$

$$A_2 = \frac{d}{ds}\left[s^3 \cdot \frac{1}{s^3 (s+1)(s+2)} \right]\Bigg|_{s=0} = \frac{d}{ds}\left[\frac{1}{(s+1)(s+2)} \right]\Bigg|_{s=0} =$$

$$= \left[-\frac{2s+3}{(s+1)^2 (s+2)^2} \right]\Bigg|_{s=0} = -\frac{3}{4}$$

$$A_1 = \frac{1}{2!}\frac{d^2}{ds^2}\left[s^3 \cdot \frac{1}{s^3 (s+1)(s+2)} \right]\Bigg|_{s=0} = \frac{1}{2!}\frac{d^2}{ds^2}\left[\frac{1}{(s+1)(s+2)} \right]\Bigg|_{s=0} =$$

$$= \frac{1}{2!}\left[\frac{2(s+1)(s+2) - 2(2s+3)^2}{(s+1)^3 (s+2)^3} \right]\Bigg|_{s=0} = \frac{1}{2} \cdot \frac{2 \cdot 2 - 2 \cdot 3^2}{2^3} = \frac{-7}{8}$$

$$\mathcal{L}\left\{e^{-\alpha t}\right\} = \frac{1}{s+\alpha}$$

$$y(t) = \mathcal{L}^{-1}\left\{\frac{-1}{s+1}\right\} + \mathcal{L}^{-1}\left\{\frac{1/8}{s+2}\right\} + \mathcal{L}^{-1}\left\{\frac{-7/8}{s}\right\} +$$

$$+ \mathcal{L}^{-1}\left\{\frac{-3/4}{s^2}\right\} + \mathcal{L}^{-1}\left\{\frac{1/2}{s^3}\right\} = -e^{-t} + \frac{1}{8}\cdot e^{-2t} - \frac{7}{8} - \frac{3}{4}\cdot t + \frac{1}{2}\cdot\frac{t^2}{2}$$

$$\mathcal{L}^{-1}\left(\frac{1}{(s+a)^{n+1}}\right) = \frac{e^{-at}\cdot t^n}{n!}$$

Cazul -c

$$P(s)=\left(s^2+as+b\right)\cdot\prod_{i=3}^{n}(s+r_i)$$

$$Z(s)=\frac{N(s)}{P(s)}=\frac{A_1 s+A_2}{s^2+as+b}+\sum_{i=3}^{n}\frac{A_i}{s+r_i}$$

A_1 si A_2 prin identificare

A_i – conform procedurilor anterioare

Exemplu – cazul c

$$Y(s)=\frac{2}{s\left(s^2+s+2\right)}=\frac{A_3}{s}+\frac{A_1 s+A_2}{s^2+s+2}$$

$$(A_3+A_1)s^2+(A_3+A_2)s+2A_3=2$$

$$A_3=1$$

$$A_3+A_2=0\Rightarrow A_2=-1$$

$$A_3+A_1=0\Rightarrow A_1=-1$$

$$Y(s)=\frac{2}{s\left(s^2+s+2\right)}=\frac{1}{s}+\frac{-s-1}{s^2+s+2}$$

$$\mathcal{L}\left\{e^{-at}\left(A\cos\omega t + B\sin\omega t\right)\right\} = \frac{A(s+a)+B\omega}{(s+a)^2+\omega^2}$$

$$y(t) = \mathcal{L}^{-1}\left\{\frac{1}{s}\right\} - \mathcal{L}^{-1}\left\{\frac{s+1}{s^2+s+2}\right\} = 1 - \mathcal{L}^{-1}\left\{\frac{(s+\frac{1}{2}) + \frac{\sqrt{3}}{3}\cdot\frac{\sqrt{3}}{2}}{\left(s+\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2}\right\} =$$

$$= 1 - e^{-\frac{1}{2}t}\cdot\left(\cos\frac{\sqrt{3}}{2}t + \frac{\sqrt{3}}{3}\sin\frac{\sqrt{3}}{2}t\right)$$