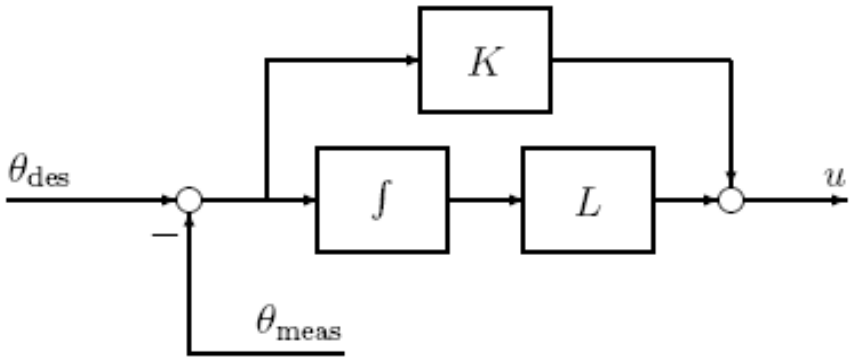
Departamentul de MECATRONICĂ
Facultatea de MECANICĂ
UNIVERSITATEA POLITHENICA TIMIŞOARA

TEORIA SISTEMELOR AUTOMATE



Prof. dr. ing. Valer DOLGA,

Cuprins_6
Functia de transfer si algebra schemelor bloc

Functia de transfer. Definire

$$y^{(n)}(t)+a_{n-1}y^{(n-1)}(t)+\ldots+a_0y(t)=$$

$$=b_mu^{(m)}(t)+b_{m-1}u^{(m-1)}(t)+\ldots b_0u(t)$$

$$y^{(k)}(t)=\frac{d^ky}{dt^k}\quad k=1,2,\ldots,n$$

$$u^{(i)}(t)=\frac{d^iu}{dt^i}\quad i=1,2,\ldots,m$$

$$y^{(k)}(t)=0\ \forall k<n$$

$$u^{(i)}(t)=0\ \ \forall i<m$$

conditii initiale

$\mathcal{L}$

$$s^nY(s)+a_{n-1}s^{n-1}Y(s)+\ldots+a_0Y(s)=$$

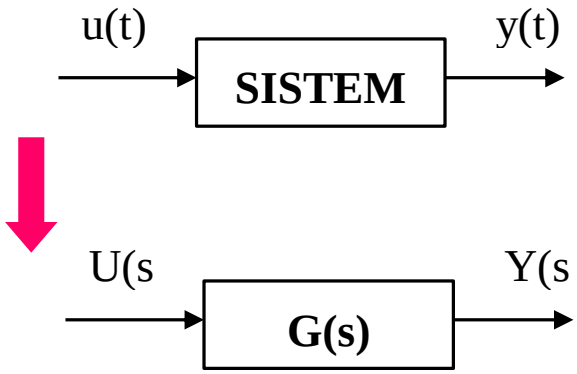
$$=b_ms^mU(s)+b_{m-1}s^{m-1}U(s)+\ldots b_0U(s)$$

$$Y(s) = \frac{b_m s^m + b_{m-1} s^{m-1} + \ldots + b_0}{s^n + a_{n-1} s^{n-1} + \ldots + a_0} U(s)$$

$$Y(s) = G(s)U(s)$$

$$G(s) = \frac{Y(s)}{U(s)}$$

$$G(s) = \frac{Y(s)}{U(s)} = \frac{b_m s^m + b_{m-1} s^{m-1} + \ldots + b_0}{s^n + a_{n-1} s^{n-1} + \ldots + a_0}$$



Ordinul sistemului

Max {n,m} ➡ ordinul sistemului

$a_0 y(t) = b_0 u(t)$ - sistem de ordinul "0"

$\mathcal{L}$

$$a_0 Y(s) = b_0 U(s)$$

$$G(s) = \frac{Y(s)}{U(s)} = \frac{b_0}{a_0}$$

$a_1 \frac{dy(t)}{dt} + a_0 y(t) = b_0 u(t)$ - sistem de ordinul "1"

$\mathcal{L}$

$$a_1 sY(s) + a_0 Y(s) = b_0 U(s)$$

$$(a_1 s + a_0) Y(s) = b_0 U(s)$$

$$G(s) = \frac{Y(s)}{U(s)} = \frac{b_0}{a_1 s + a_0}$$

$$M \cdot \frac{d^2 y(t)}{dt^2} + K_d \cdot \frac{dy(t)}{dt} + K \cdot y(t) = F(t)$$

- ordinul sistemului - ?
- functia de transfer - ?

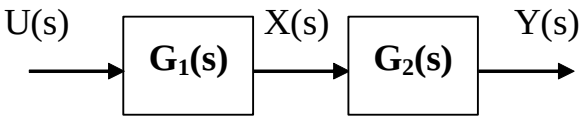
Studiu individual

Algebra schemelor bloc

Transmisia mecanica = sistemul_2

Motor electric = sistemul_1

$\mathcal{L}$

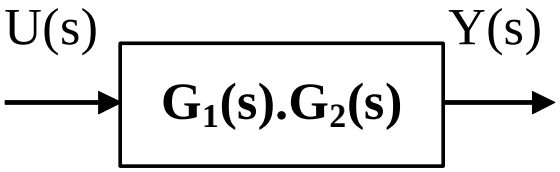


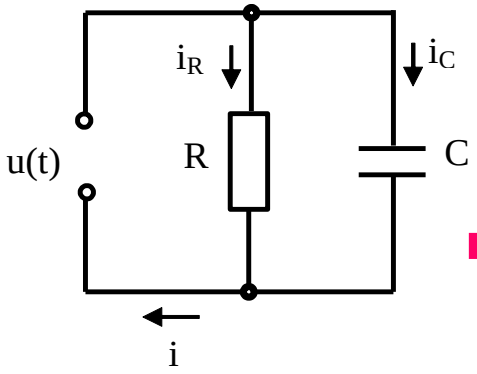
$$X(s) = G_1(s)U(s)$$

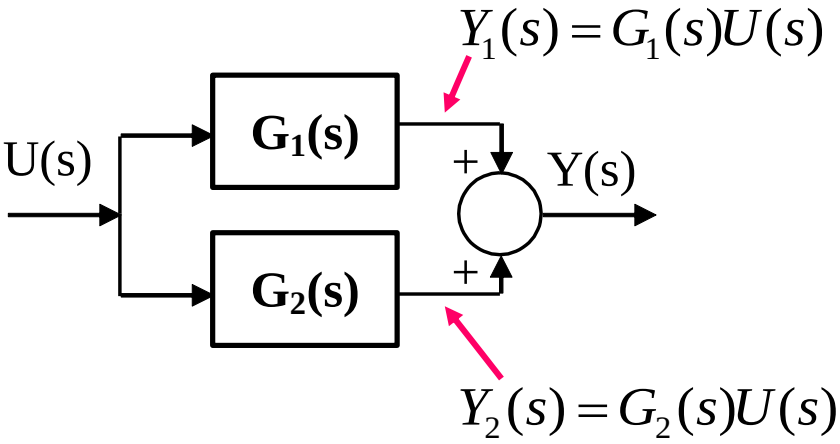
$$Y(s) = G_2(s)X(s)$$

$$Y(s) = G_1(s)G_2(s)U(s)$$

$$G(s) = G_1(s) \cdot G_2(s)$$



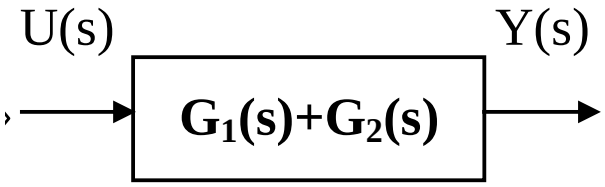


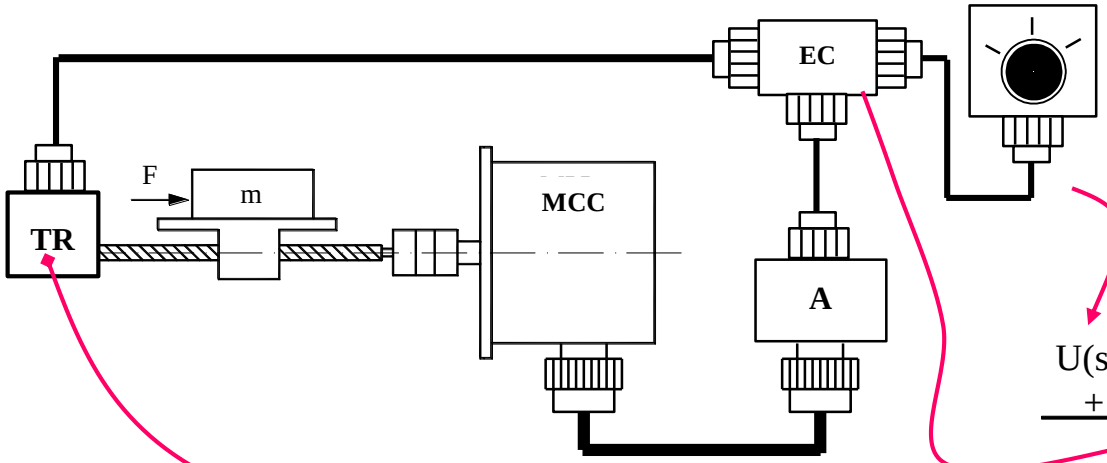


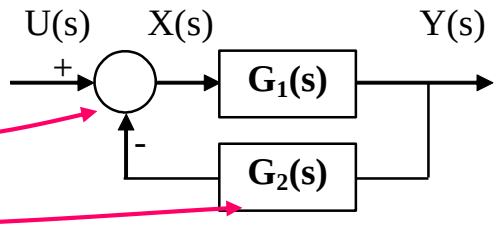
$$Y(s) = G_1(s)U(s) + G_2(s)U(s) = \left[G_1(s) + G_2(s)\right]U(s)$$

$$Y(s) = G(s) \cdot U(s)$$

$$G(s) = G_1(s) + G_2(s)$$





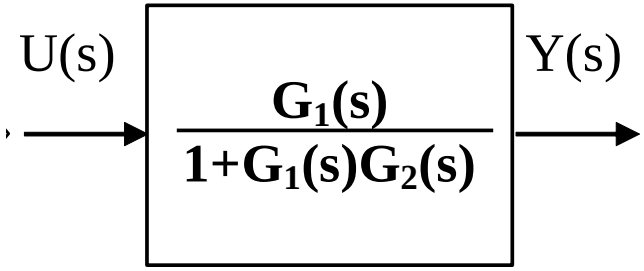


$$Y(s) = G_1(s)X(s)$$

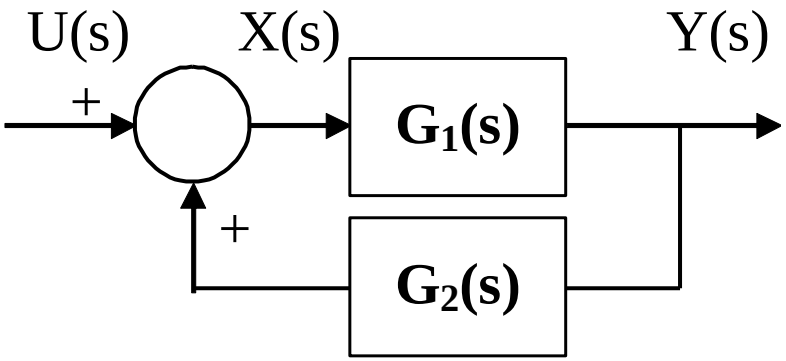
$$X(s) = U(s) - G_2(s)Y(s)$$

$$Y(s) = G_1(s)\left[U(s) - G_2(s)Y(s)\right]$$

$$Y(s) = \frac{G_1(s)}{1 + G_1(s)G_2(s)}U(s)$$



$$G(s) = \frac{G_1(s)}{1 + G_1(s)G_2(s)}$$



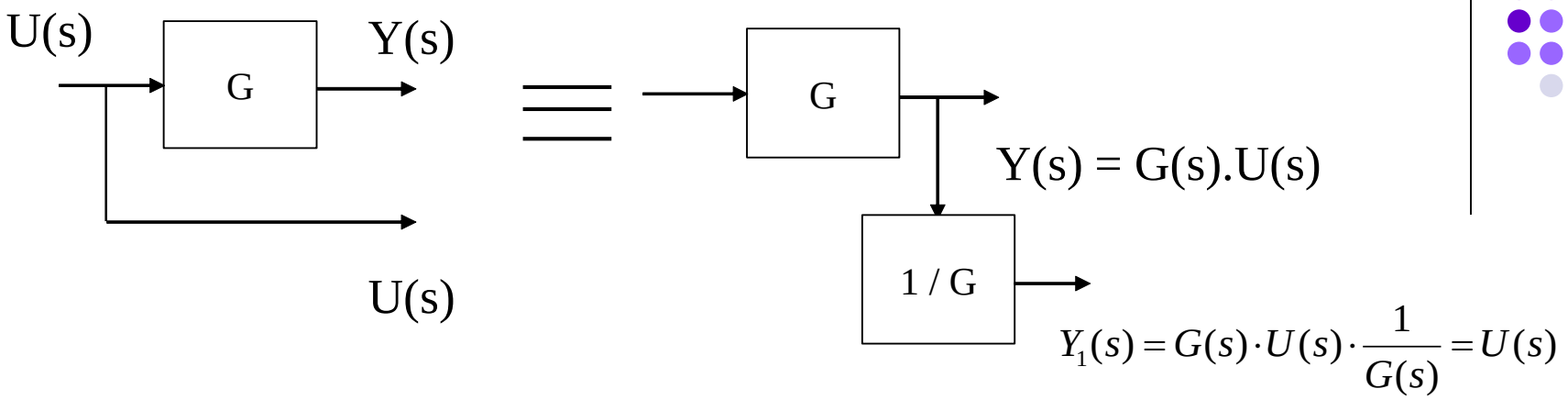
$$Y(s) = G_1(s)X(s)$$

$$X(s) = U(s) + G_2(s)Y(s)$$

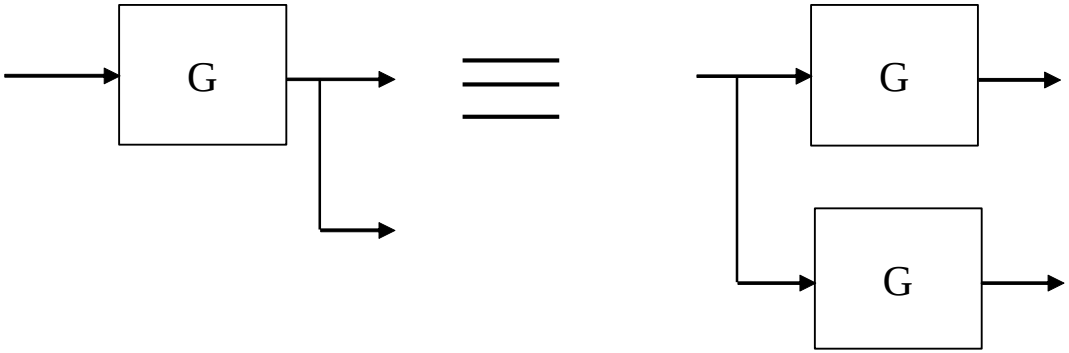
$$Y(s) = G_1(s)\left[U(s) + G_2(s)Y(s)\right]$$

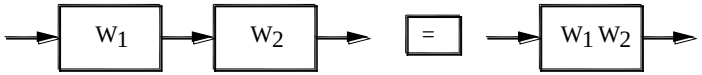
$$Y(s) = \frac{G_1(s)}{1 - G_1(s)G_2(s)}U(s)$$

$$G(s) = \frac{G_1(s)}{1 - G_1(s)G_2(s)}$$

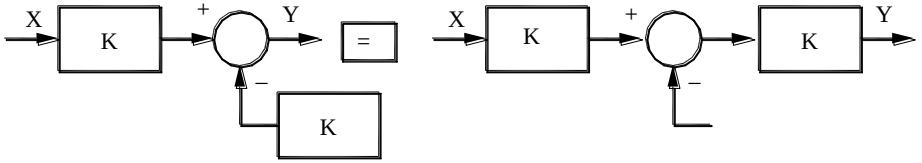


$$Y_1(s) = G(s) \cdot U(s) \cdot \frac{1}{G(s)} = U(s)$$

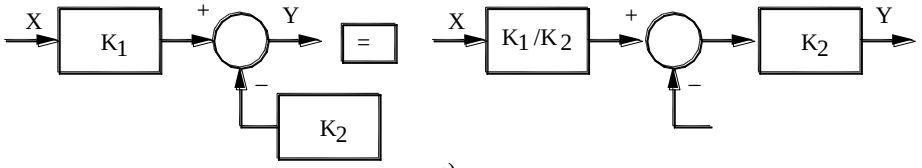



W1
W2
=
W1 W2

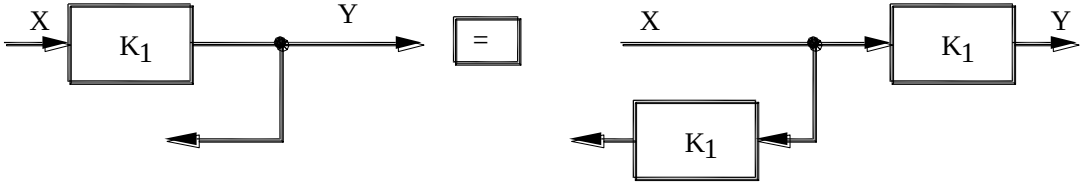
a)


X
K
+
Y
=
K
X
K
+
K
Y

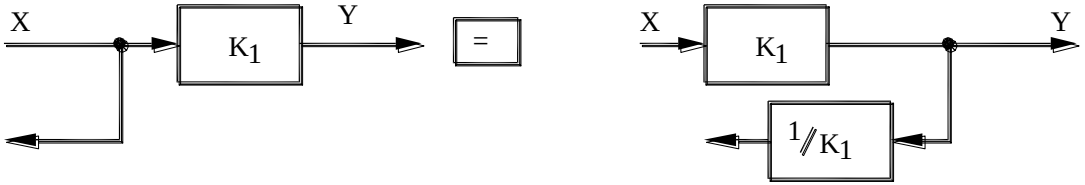
b)


X
K1
+
Y
=
K2
X
K1 /K2
+
K2
Y

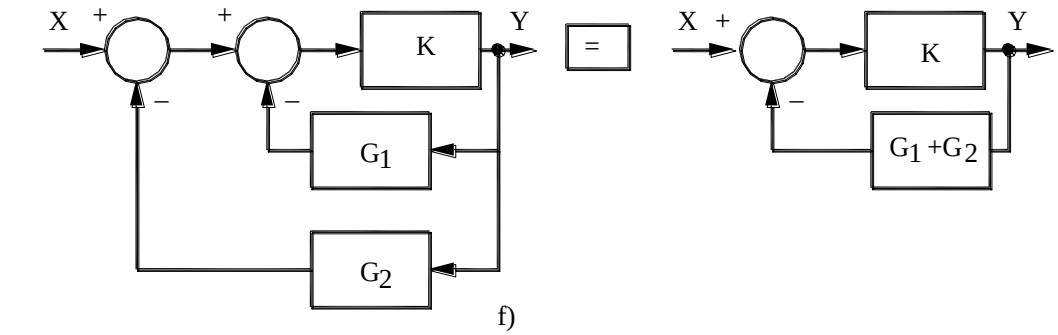
c)


X
K1
Y
=
X
K1
Y
K1

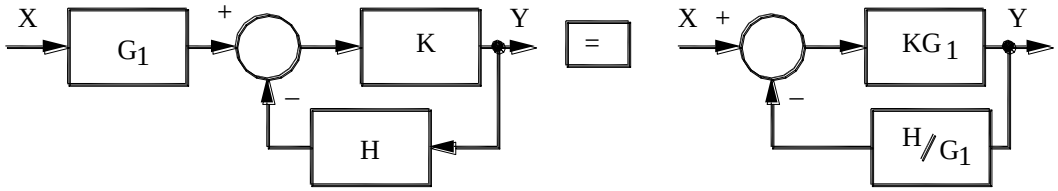
d)


X
K1
Y
=
X
K1
Y
1/K1

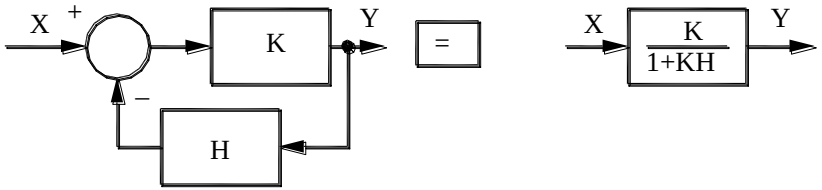
e)



f)

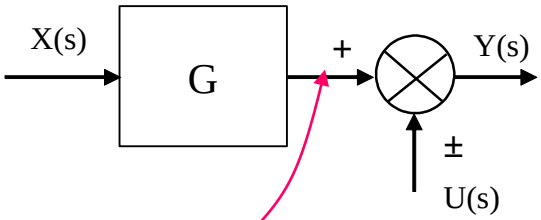


g)



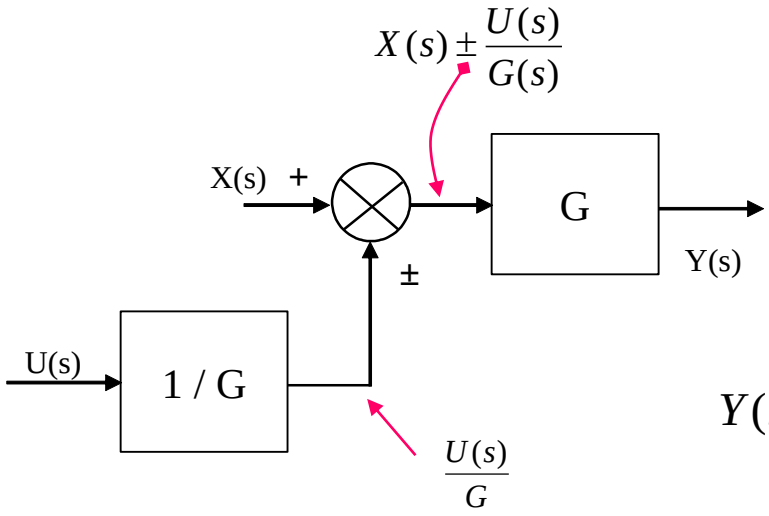
h)

$$Y(s) = Y_1(s) \pm U(s) = G(s) \cdot X(s) \pm U(s)$$



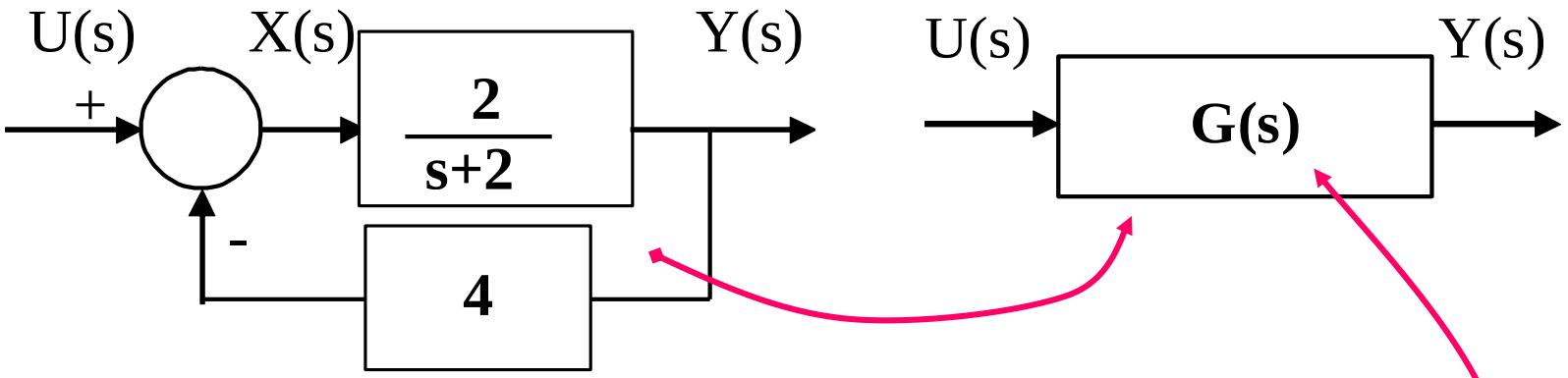
$$Y_1(s) = G(s) \cdot X(s)$$

≡



$$Y(s) = G(s) \cdot \left(X(s) \pm \frac{U(s)}{G(s)} \right) = G(s) \cdot X(s) \pm U(s)$$

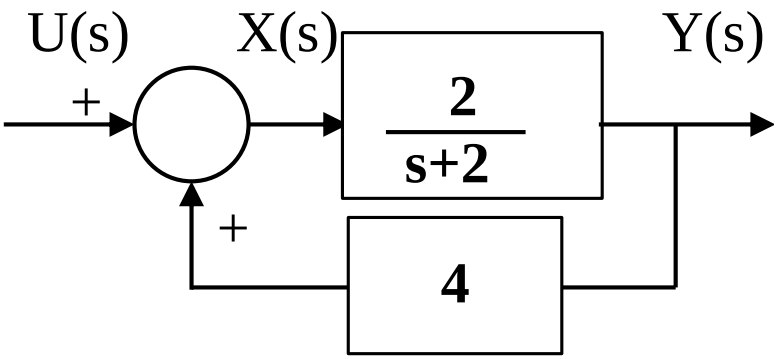
Exemplu de calcul



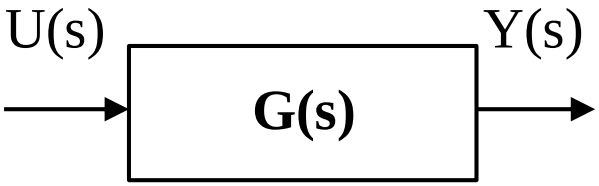
$$G_1(s) = \frac{2}{s+2} \qquad G_2(s) = 4$$

$$G(s) = \frac{G_1(s)}{1 + G_1(s)G_2(s)} = \frac{\frac{2}{s+2}}{1 + 4 \cdot \frac{2}{s+2}} = \frac{\frac{2}{s+2}}{\frac{s+2+8}{s+2}} = \frac{2}{s+10}$$

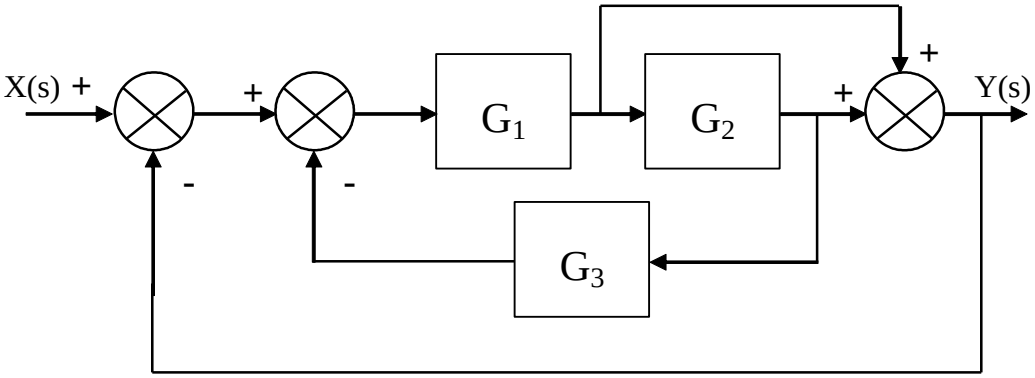
Exemplu_studiu

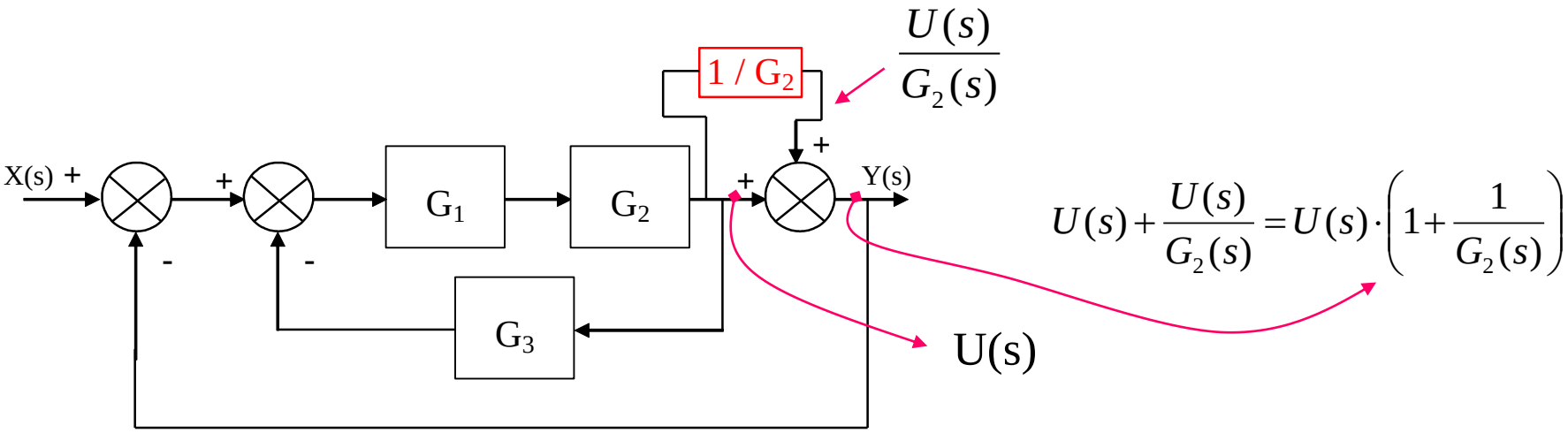


?

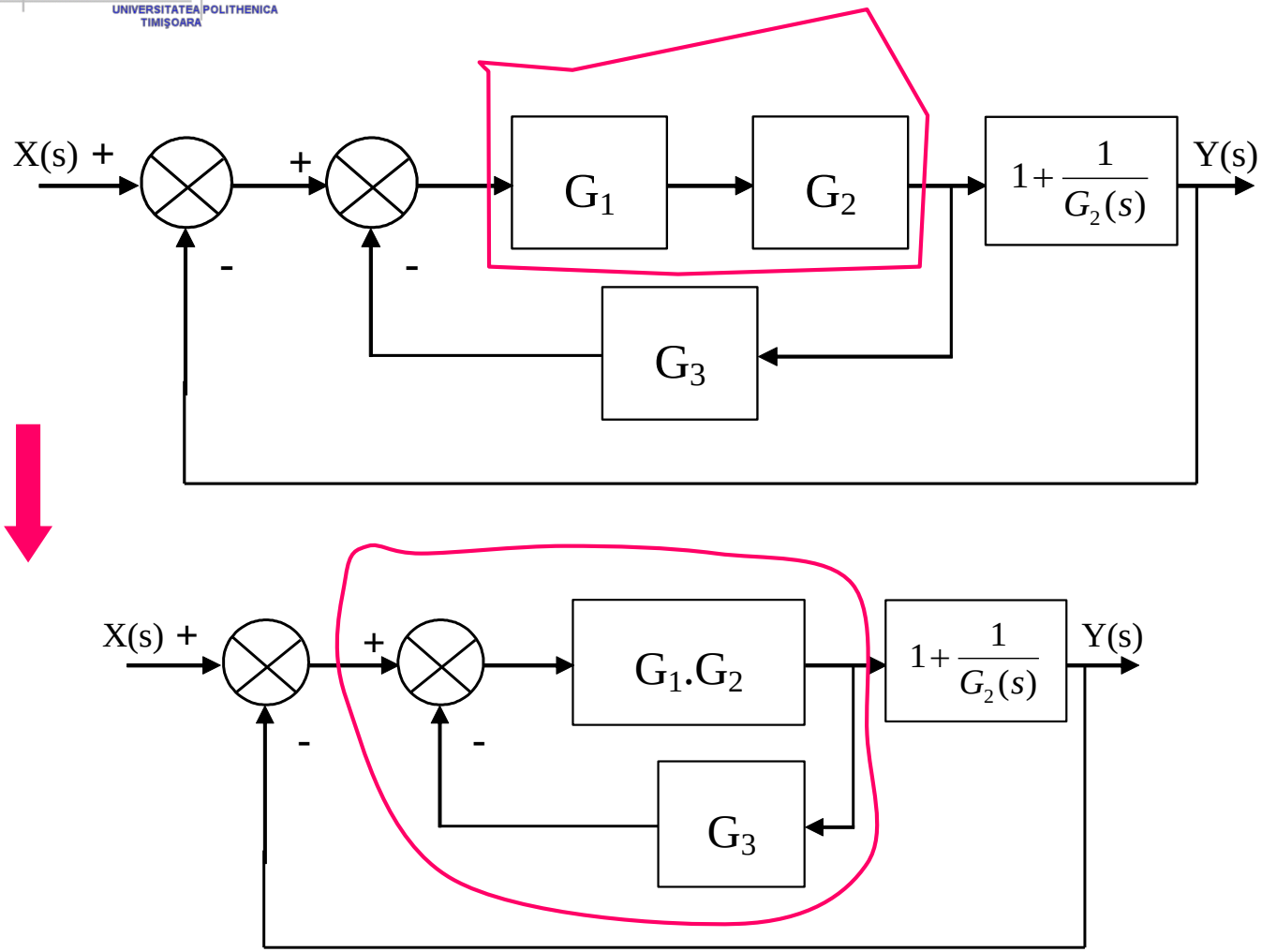


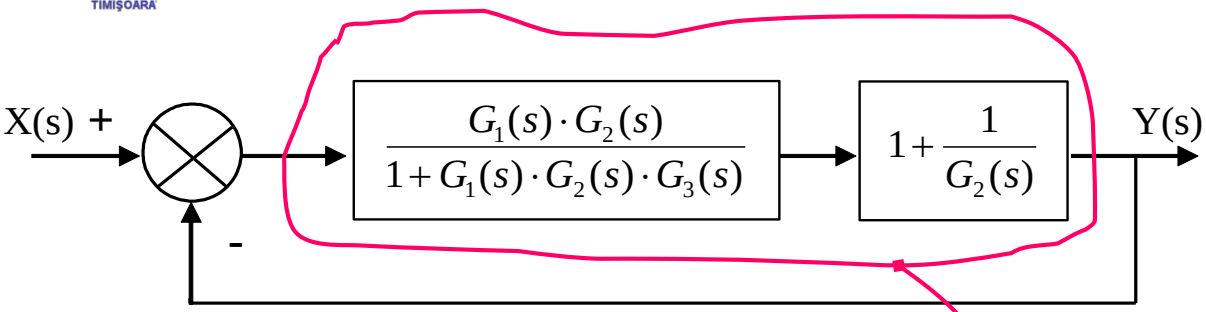
Exemplu de calcul



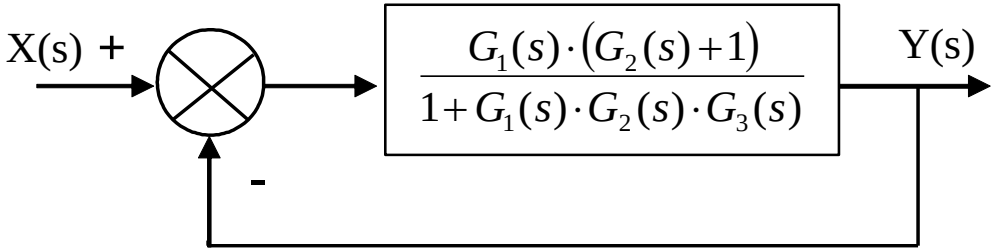


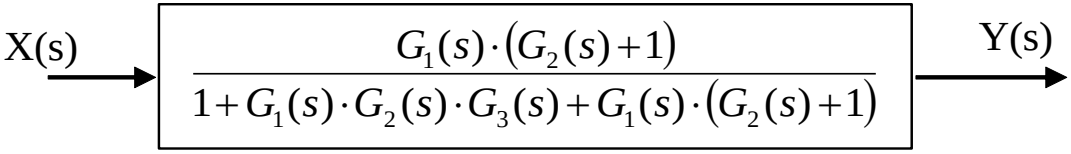
$$U(s)+\frac{U(s)}{G_2(s)}=U(s)\cdot\left(1+\frac{1}{G_2(s)}\right)$$


X(s)
+
+
-
-
G1
G2
1+ 1/G2(s)
Y(s)
G3
X(s)
+
+
-
-
G1.G2
1+ 1/G2(s)
Y(s)
G3

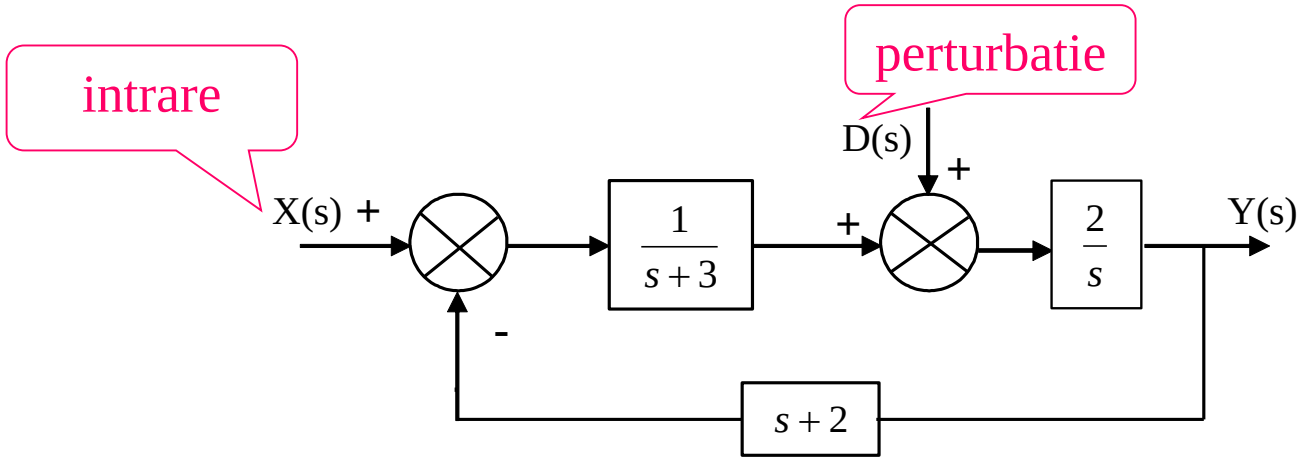


$$\frac{G_1(s)\cdot G_2(s)}{1+G_1(s)\cdot G_2(s)\cdot G_3(s)}\cdot\frac{G_2(s)+1}{G_2(s)}$$



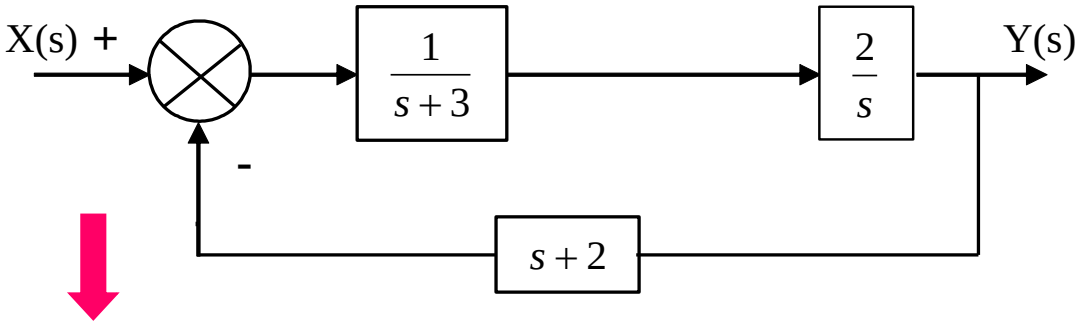


Sistem cu intrari multiple si o singura iesire (MISO)



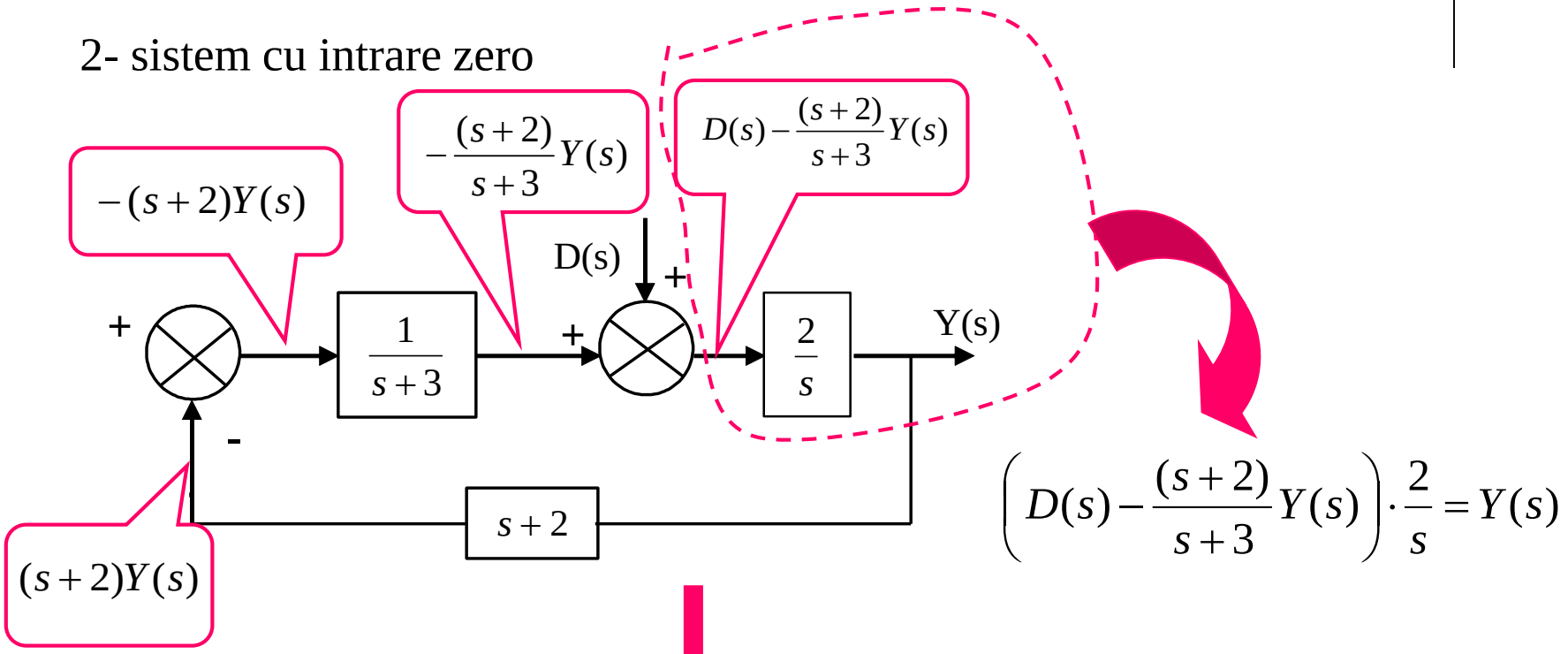
Principiul superpozitiei

1 – sistem cu perturbatie zero



$$G(s)=\frac{Y(s)}{X(s)}=\frac{\frac{2}{s}\cdot\frac{1}{s+3}}{1+\frac{2}{s}\cdot\frac{1}{s+3}\cdot(s+1)}=\frac{\frac{2}{s(s+3)}}{\frac{s(s+3)+2(s+1)}{s(s+3)}}=\frac{2}{s(s+3)+2(s+1)}$$

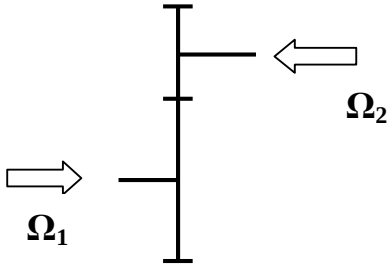
2- sistem cu intrare zero



$$\left(D(s)-\frac{(s+2)}{s+3}Y(s)\right)\cdot\frac{2}{s}=Y(s)$$

$$Y(s)=\frac{2}{s(s+3)+2(s+1)}\cdot X(s)+\frac{2(s+3)}{s(s+3)+2(s+1)}\cdot D(s)$$

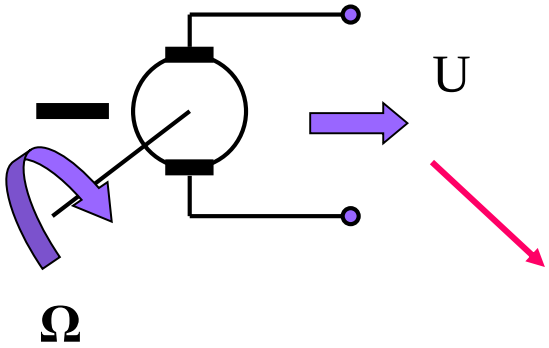
Componente de sistem si functii de transfer



$$i = \frac{\Omega_1}{\Omega_2} = \frac{z_2}{z_1} \quad \Rightarrow \quad G(s) = \frac{1}{i}$$

$$\eta = \frac{P_u}{P_C} \quad \Rightarrow \quad G(s) = \frac{M_2}{M_1} = i\eta$$

$$U = K\Omega \quad \Rightarrow \quad G(s) = \frac{U}{\Omega} = K$$



$$e = Ri + L \cdot \frac{di}{dt}$$

$$k\Omega = u + \frac{L}{R}\frac{du}{dt}$$

$$k\Omega(s) = U(s) + s\frac{L}{R}U(s)$$

$$k\Omega(s) = \left(1 + s\frac{L}{R}\right)U(s)$$

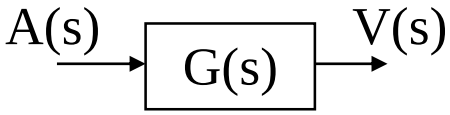
$$G(s) = \frac{U(s)}{\Omega(s)} = \frac{k}{s\frac{L}{R}+1} = \frac{k}{\tau s+1}$$

$$a = a(t)$$

$$v = \int a\,dt$$

$$V(s) = \frac{1}{s} A(s)$$

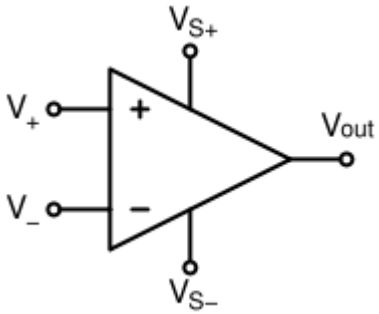
$$G(s) = \frac{V(s)}{A(s)} = \frac{1}{s}$$



$$s = \int v\,dt$$

$$S(s) = sV(s)$$

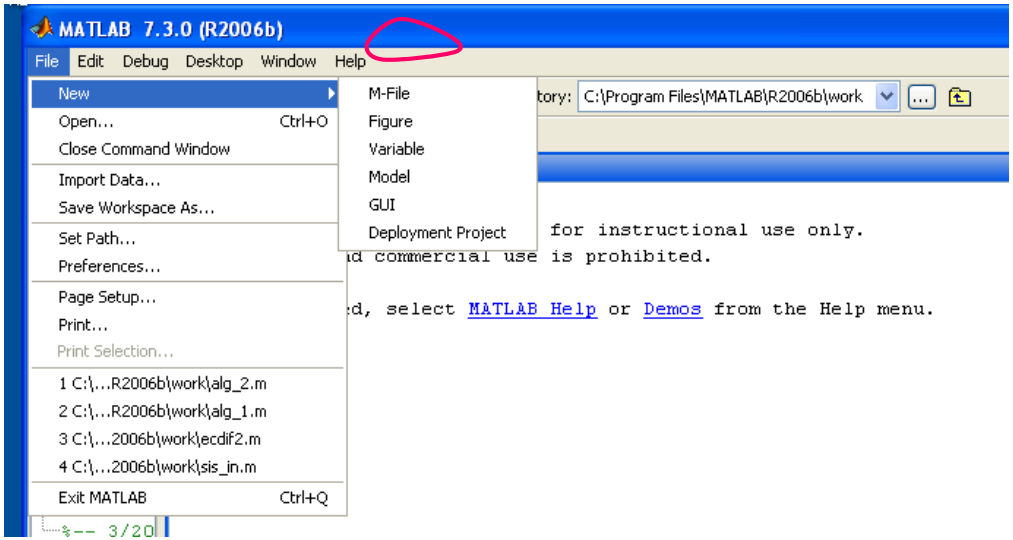
$$G(s) = \frac{S(s)}{V(s)} = s$$

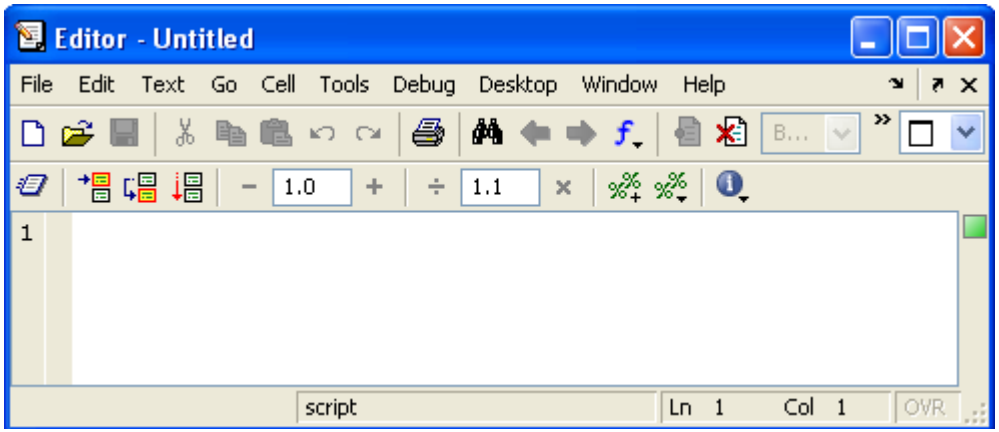


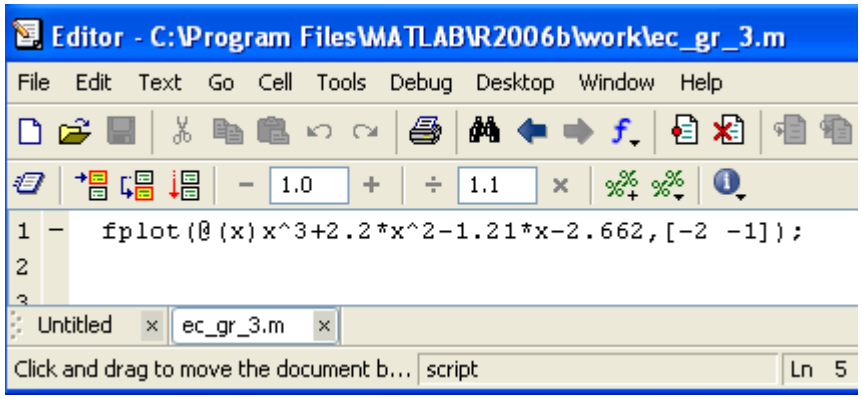
$$V_{out} = (V_+ - V_-) \cdot A$$

$$G(s) = \frac{V_{out}}{V_+ - V_-} = A$$

Fisiere “m” *.m


MATLAB 7.3.0 (R2006b)
File Edit Debug Desktop Window Help
New
Open... Ctrl+O
Close Command Window
Import Data...
Save Workspace As...
Set Path...
Preferences...
Page Setup...
Print...
Print Selection...
1 C:\...R2006b\work\alg_2.m
2 C:\...R2006b\work\alg_1.m
3 C:\...2006b\work\ecdif2.m
4 C:\...2006b\work\sis_in.m
Exit MATLAB Ctrl+Q
M-File
Figure
Variable
Model
GUI
Deployment Project
C:\Program Files\MATLAB\R2006b\work
for instructional use only.
commercial use is prohibited.
MATLAB Help
Demos
from the Help menu.


Editor - Untitled
File Edit Text Go Cell Tools Debug Desktop Window Help
1.0
1.1
1
script
Ln 1 Col 1
OVR


Editor - C:\Program Files\MATLAB\R2006b\work\ec_gr_3.m
File Edit Text Go Cell Tools Debug Desktop Window Help
1.0
1.1
1 – fplot(@(x)x^3+2.2*x^2-1.21*x-2.662,[-2 -1]);
2
3
Untitled
ec_gr_3.m
Click and drag to move the document b...
script
Ln 5

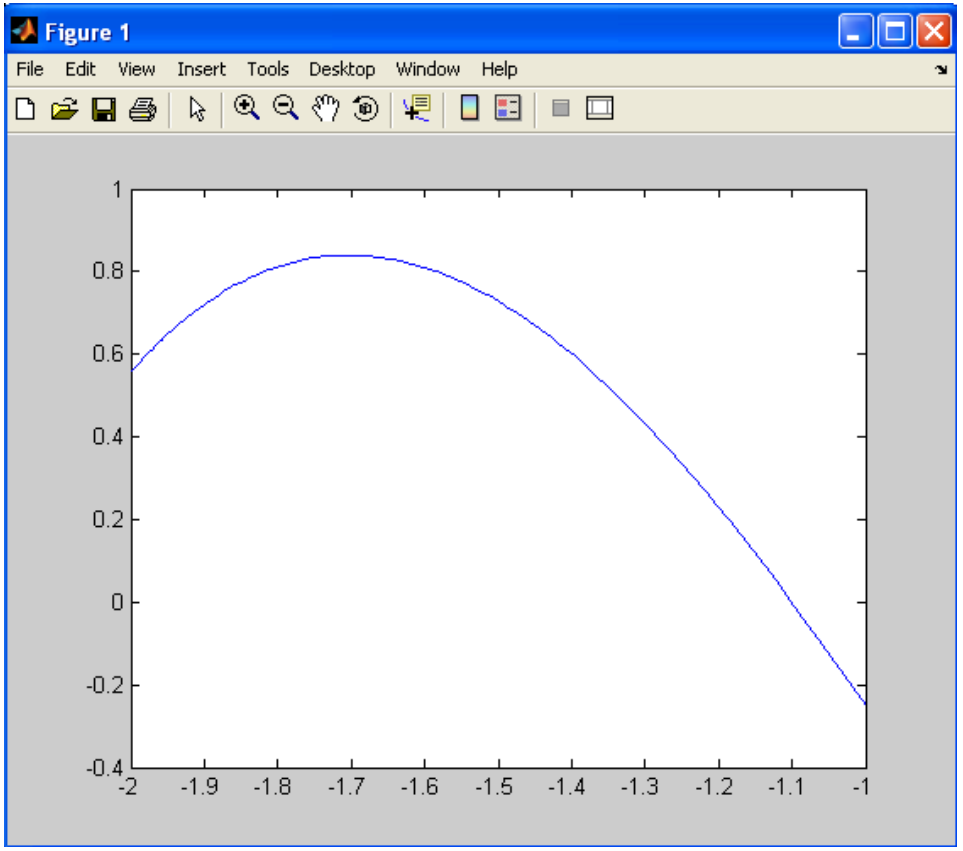


Figure 1
File Edit View Insert Tools Desktop Window Help
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-2 -1.9 -1.8 -1.7 -1.6 -1.5 -1.4 -1.3 -1.2 -1.1 -1

Poli si zerouri

$$G(s) = \frac{Q(s)}{P(s)}$$

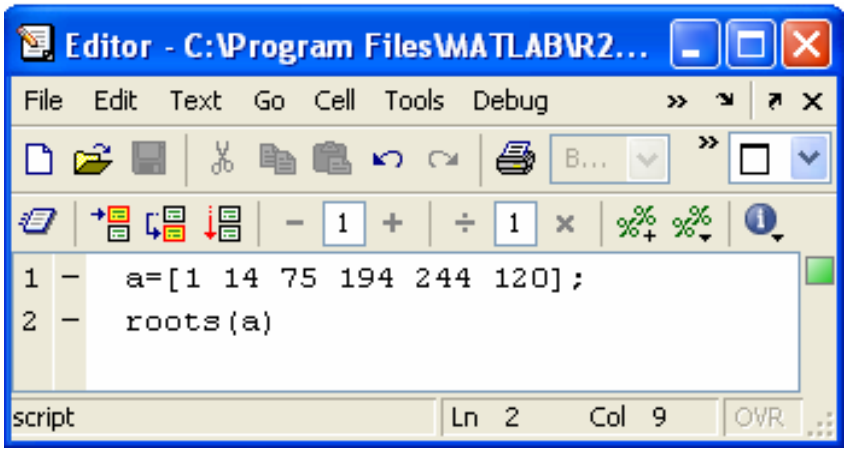
$Q(s) = 0 \rightarrow s_1, s_2, ... s_n$ = **zerourile functiei de transfer**

$P(s) = 0 \rightarrow p_1, p_2, ... p_n$ = **polii functiei de transfer**

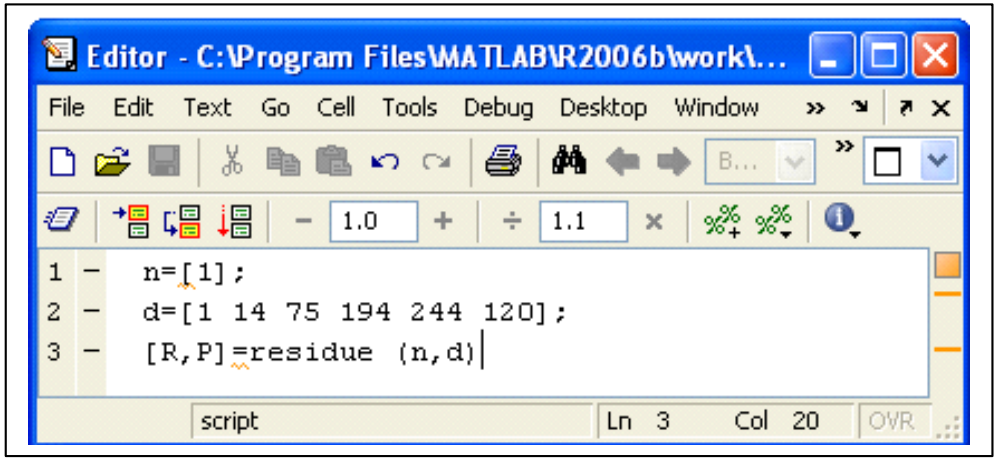
$$G(s) = \frac{1}{s^5 + 14s^4 + 75s^3 + 194s^2 + 244s + 120}$$

polii functiei de transfer

$$G(s) = \frac{1}{(s-p_1)\cdot(s-p_2)\cdot(s-p_3)\cdot(s-p_4)\cdot(s-p_5)} = \frac{a_1}{s-p_1} + \frac{a_2}{s-p_2} + \frac{a_3}{s-p_3} + \frac{a_4}{s-p_4} + \frac{a_5}{s-p_5}$$




```
ans =

   -5.0000
   -3.0000
   -2.0000
   -2.0000 + 0.0000i
   -2.0000 - 0.0000i
```

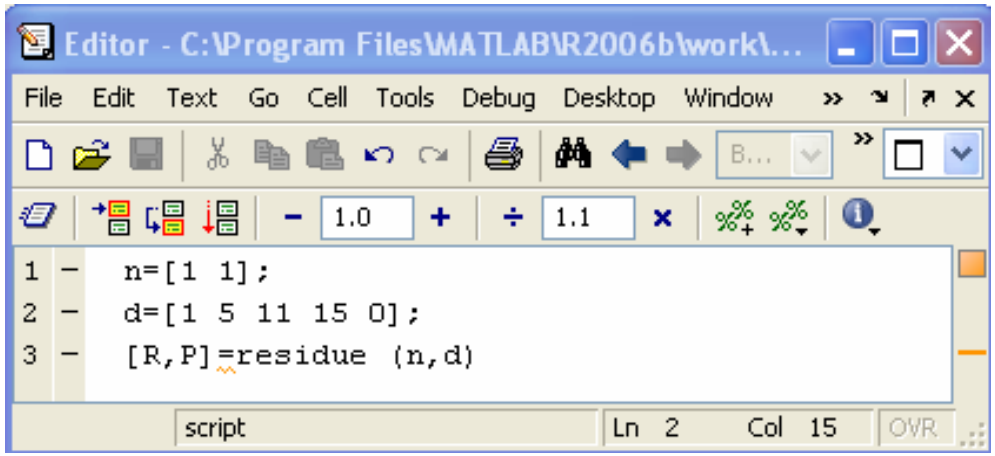



```
R =

    0.0185
   -0.5000
    0.4815
   -0.4444
    0.3333


P =

   -5.0000
   -3.0000
   -2.0000
   -2.0000
   -2.0000
```

$s_1 = -1$ **un zero !**

$$G(s) = \frac{s+1}{s^4 + 5s^3 + 11s^2 + 15s}$$

4 poli !




```
R =

    0.0833
   -0.0750 - 0.0250i
   -0.0750 + 0.0250i
    0.0667


P =

   -3.0000
   -1.0000 + 2.0000i
   -1.0000 - 2.0000i
         0
```

$$G(s) = \frac{s+1}{s^4 + 5s^3 + 11s^2 + 15s} = \frac{0.0833}{s+3} + \frac{-0.075 - 0.025i}{s+1-2i} + \frac{-0.075 + 0.025i}{s+1+2i} + \frac{0.0667}{s} =$$

$$= \frac{0.0833}{s+3} - \frac{0.15s + 0.05}{s^2 + 2s + 5} + \frac{0.0667}{s}$$