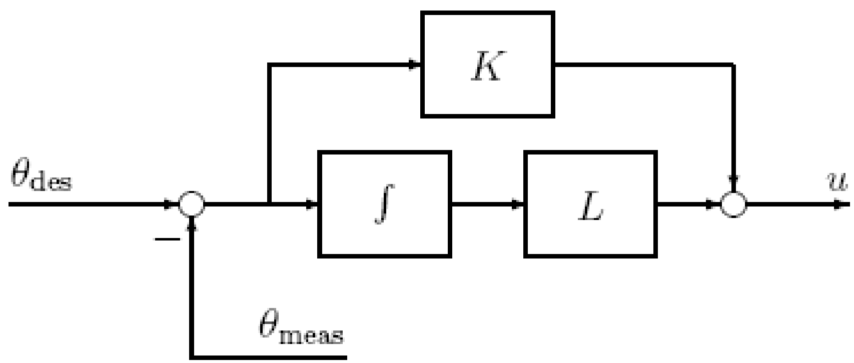
Departamentul de MECATRONICĂ
Facultatea de MECANICĂ
UNIVERSITATEA POLITHENICA TIMIŞOARA

TEORIA SISTEMELOR AUTOMATE



Prof. dr. ing. Valer DOLGA,

Cuprins_12

Raspunsul la frecventa

Comportarea la frecventa: - interes:

- Identificarea experimentala a parametrilor
- Analiza si sinteza sistemelor automate

O functie **f(t) – periodica, nesinusoidala, continua…-** poate fi descompusa in serie Fourier:

$$f(t) = a_0 + \sum_{1}^{\infty}\left(a_k \cos k\omega t + b_k \sin k\omega t\right)$$

$$\omega = \frac{2\pi}{T} \; - pulsatia$$

$$a_0, a_k, b_k \; - coeficienti\ ai\ seriei\ Fourier$$

Integrala Fourier

$$f(t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} F(j\omega) e^{j\omega t} d\omega$$

$$F(j\omega) = \int_{-\infty}^{+\infty} f(t) e^{-j\omega t} dt$$ = transformata Fourier a functiei f(t)

$$F(s) = \int_{0}^{+\infty} f(t) e^{-st} dt = \mathcal{L}[f(t)]$$ transformata Laplace

- relatiile sunt identice daca:

σ = 0 si $s = j\omega$

- **daca se cunosc perechile de functii f(t) – F(s) se obtin perechile de functii f(t) – F(jω) prin simpla inlocuire s → jω**

Cunoscind functia de transfer al unui element se poate obtine – **raspunsul la frecventa:**

$$G(j\omega) = \frac{Y(j\omega)}{X(j\omega)} = \frac{Q(j\omega)}{P(j\omega)} = \frac{b_m (j\omega)^m + b_{m-1}(j\omega)^{m-1} + \ldots b_0}{(j\omega)^n + a_{n-1}(j\omega)^{n-1} + \ldots a_0}$$

• reprezentarea in planul complex a extremitatii fazorului G(jω) – loc de transfer sau caracteristica amplitudine - faza

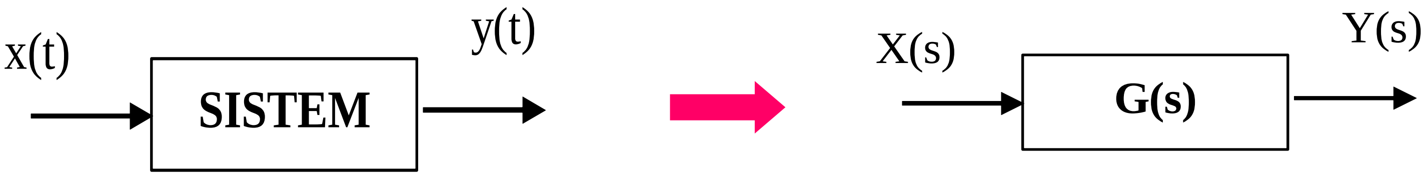
• raspunsul la frecventa se exprima prin:

$$G(j\omega) = G(\omega)e^{j\varphi(\omega)} = U(\omega) + jV(\omega)$$

$$G(\omega) = |G(j\omega)| = \sqrt{U^2(\omega) + V^2(\omega)} > 0$$

- G(ω) – reprezentata in planul G – ω poarta numele de caracteristica modul – frecventa
- U(ω) si V(ω) reprezentate in acelasi plan poarta numele de caracteristica reala, respectiv imaginara, de frecventa
- φ(ω) – caracteristica faza - frecventa

Exemplul_1



$$G(s) = \frac{5}{s+2}$$

$$G(j\omega) = \frac{5}{2 + j\omega}$$

$$G(j\omega) = \frac{5}{2 + j\omega} \cdot \frac{2 - j\omega}{2 - j\omega} = \frac{10 - j5\omega}{4 + \omega^2} = \underbrace{\frac{10}{4 + \omega^2}}_{U(\omega)} - j\underbrace{\frac{5\omega}{4 + \omega^2}}_{V(\omega)}$$

Calculati modulul si faza pentru G(jω)

Exemplul_2

$$G(s)=\frac{2}{s+1}$$

$$G(j\omega)=\frac{2}{1+j\omega}$$

$$G(j\omega)=\frac{2}{1+j\omega}\cdot\frac{1-j\omega}{1-j\omega}=\frac{2-j2\omega}{1+\omega^2}=\frac{2}{1+\omega^2}-j\frac{2\omega}{1+\omega^2}$$

$$|G(j\omega)|=\sqrt{\left(\frac{2}{1+\omega^2}\right)^2+\left(\frac{2\omega}{1+\omega^2}\right)^2}=\frac{2}{\sqrt{1+\omega^2}}$$

$$tg\varphi=\frac{-\frac{2\omega}{1+\omega^2}}{\frac{2}{1+\omega^2}}=-\omega$$

$$ex.\ numeric: \omega=3\ rad/s$$

$$|G(j\omega)|=\frac{2}{\sqrt{1+3^2}}=0.63$$

$$tg\varphi=-3$$

$$\varphi=-72^0$$

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Doua elemente cu $G_1(s)$si $G_2(s)$ legate in serie:

$s = j\omega$

$$G(j\omega) = G(\omega)e^{j\varphi(\omega)} = G_1(j\omega)\cdot G_2(j\omega) = G_1(\omega)\cdot G_2(\omega)e^{j[\varphi_1(\omega)+\varphi_2(\omega)]}$$

$$\lg G(\omega) = \lg G_1(\omega) + \lg G_2(\omega)$$

$$\varphi(\omega) = \varphi_1(\omega) + \varphi_2(\omega)$$

Generalizati relatiile anterioare pentru n elemente legate in serie !!

Reprezentarile logaritmice – preluate din acustica !

• *pentru evaluarea variatiei puterii N a unui semnal in raport cu puterea de referinta N_0 a fost definit* ***belul (B)***

$$QB = \lg\left(\frac{N}{N_0}\right)$$

• *s-a admis ca unitate – decibelul (dB)*

$$dB = 10\lg\left(\frac{N}{N_0}\right)$$

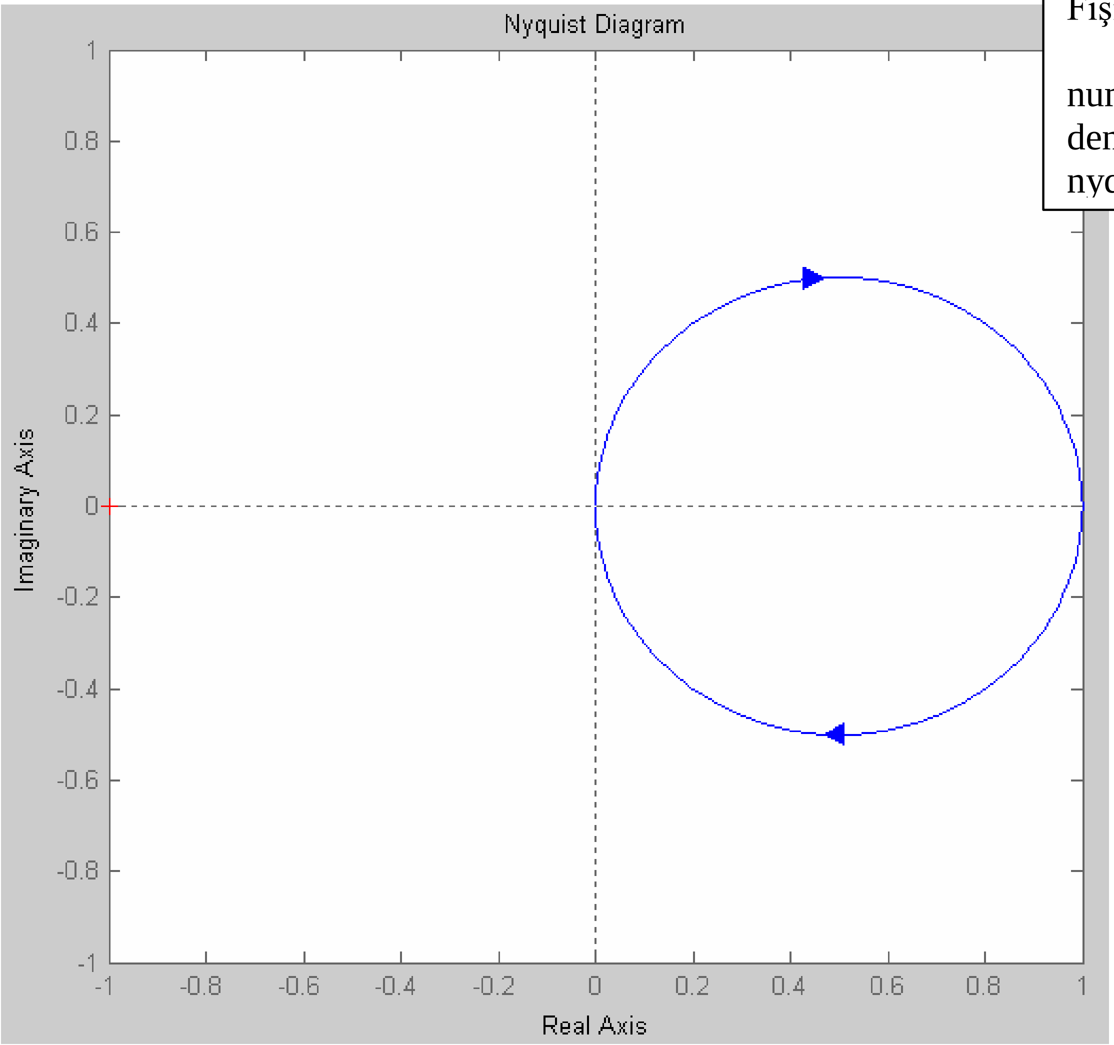
• *Daca se considera nu puterea semnalului ci o alta marime (curent, tensiune, presiune etc.) reprezentarea respecta relatia:*

$$dB = 20\lg\left(\frac{A_1}{A_2}\right)$$

• *in literatura de specialitate – caracteristicile logaritmice de frecventa =* ***diagrame BODE*** ***(pe abscisa frecventa)***

• ***diagramele Nyquist*** afişează pe acelaşi grafic atât amplitudinea, cât şi faza, utilizând frecvenţa ca şi parametru al graficului

Exemplul_3



Fişierul Matlab:

```
num=1;
den=[1 1];
nyquist(num, den)
```

$$G(j\omega) = \frac{1}{j\omega + 1}$$

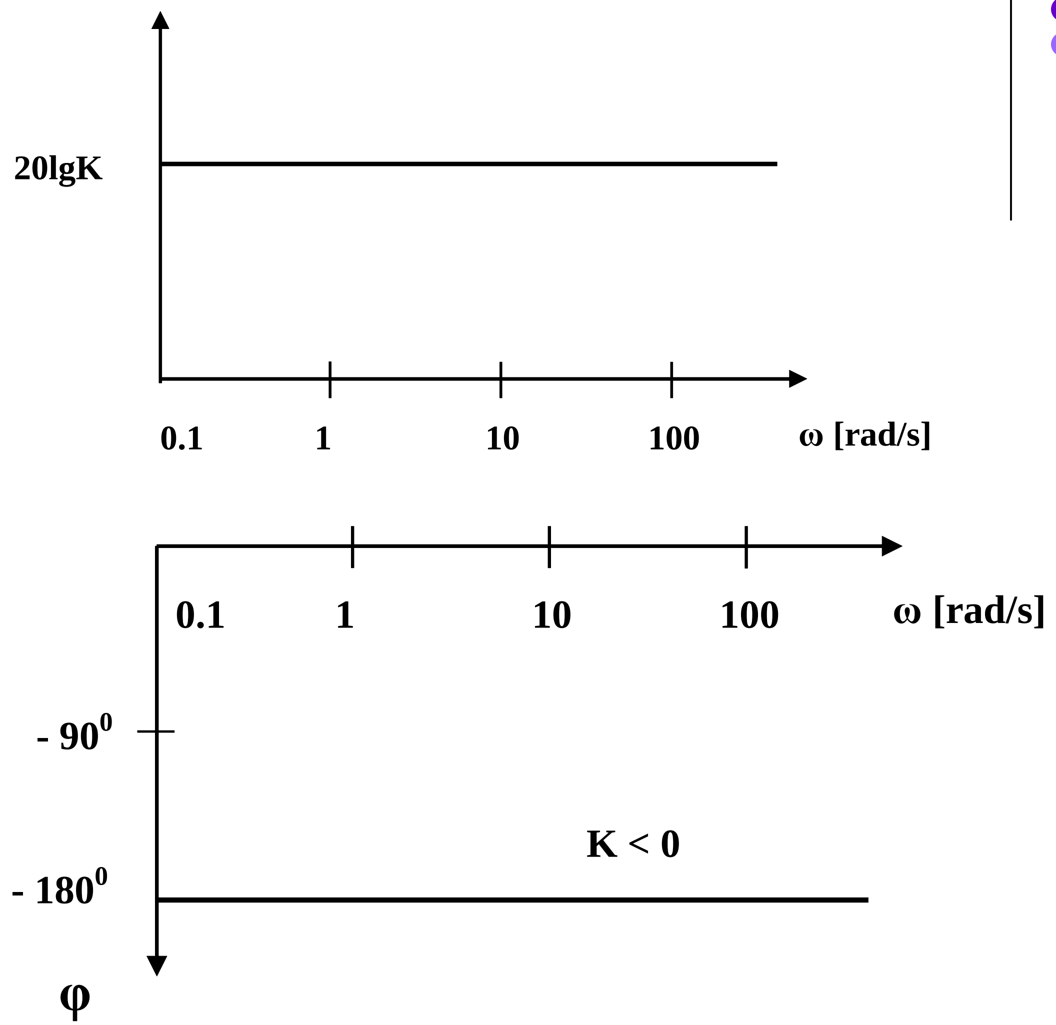
Diagramele Bode

$$G(s) = K$$

$$G(j\omega) = K$$

$$\left|G(j\omega)\right| = K$$

$$\varphi = \begin{cases} 0^0 \; daca \, K > 0 \\ -180^0 \; daca \, K < 0 \end{cases}$$



$$G(s)=\frac{1}{s}$$

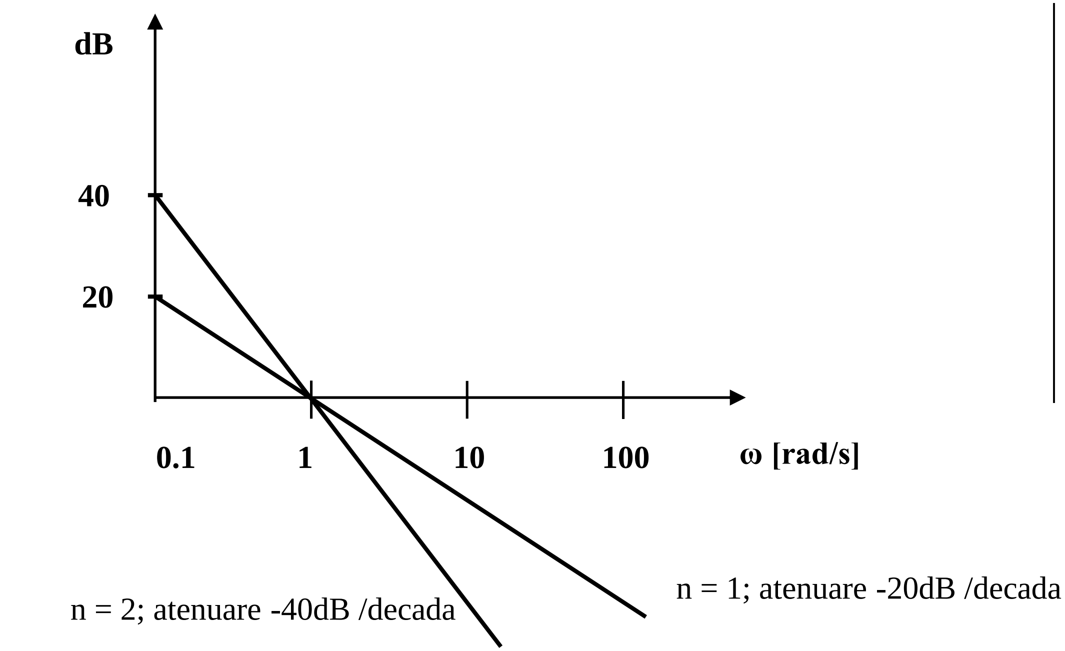
$$G(j\omega)=\frac{1}{j\omega}=-\frac{j}{\omega}$$

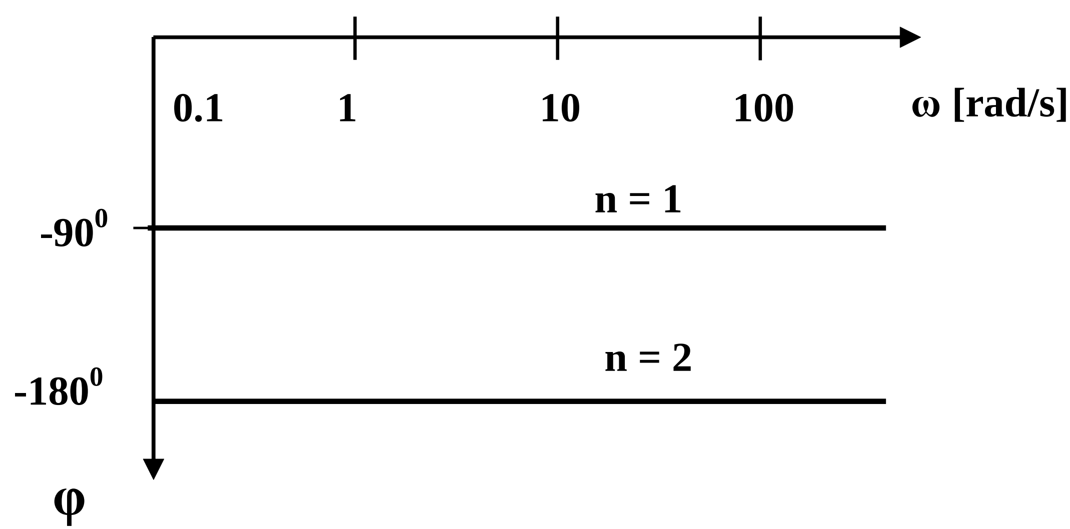
$$\left|G(j\omega)\right|=20\lg\left(\frac{1}{\omega}\right)=-20\lg(\omega)$$

$$tg\varphi=\frac{-\frac{1}{\omega}}{0}=-\infty$$

$$\varphi=-90^{0}$$

$$ex.numeric\,\omega=1rad/s$$





Exemplu_4

$$G(s) = \frac{s+3}{s^3 + 2s^2 + 3s + 4}$$

Fisier.m

bode([1 3],[1 2 3 4])